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Prediction of filter cake resistance

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PREDICTION OF FILTER CAKE RESISTANCE

by

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INTRODUCTION

General

The desire of man to improve the quality of water is not new. It is, probably, as old as the man himself. Evidence of this is found in a quotation taken from a body of medical lore in Sanskrit said to date from 2000 B.C. (1).

> "Impure water should be purified by filtration through sand and coarse gravel, by being heated in the sun, or by dipping a heated iron into it and then allowed to cool."

Another evidence available is in the instructions described in "Ousruta Sanghita" and date back to pre-Christian era (2).

> "It is good to keep water in copper vessels, to expose it to sunlight and filter through charcoal."

History provides many examples of man's efforts to treat water. The raw water, however, has continuously deteriorated in quality throughout history. Whereas, the prehistoric or caveman and the ancient man had relatively clean water to treat, man today is not very fortunate in that respect. The sources of raw water, such as streams and lakes, are far more polluted today than ever before. With the continuous discharge of human wastes and the industrial wastes into our subsurface and surface waters, there appears to be no end to raw water degradation. Industrial wastes are becoming more complex and more toxic every day. The discharge of

radioactive wastes only compounds an already complex problem. With expanding population and ever growing industry in all parts of the world, the magnitude of the problem can only grow, perhaps beyond manageable proportions. Who is to blame for this increasing pollution? Man and his activities.

Still another factor adds to the gravity of this situation. Today the higher living standards we require demand that the desired "water quality standards" be far superior today than it has been at any time in the past. Our sense of cleanliness and our awareness of it, are perhaps the greatest today. The improved living conditions have made people less resistant to disease; which only increases the need for a safer water.

One possible alternative to the solution of this big problem lies in a cessation of all mans activities; perhaps reversal in most cases. Man might have to go back to his cave to achieve that objective ultimately. The most plausible alternative lies in accepting the challenge to invent new methods, and to improve the existing ones, for water and waste water treatment. Only the second alternative would appeal to most people.

The increasingly complex raw water needs to be subjected to a series of treatment steps before it is fit for consumption; the degree of treatment in all cases would depend on the particular usage. No single treatment will ever replace

the series of treatments currently adopted, although some may be eliminated and others improved or modified with continued research in the future. Filtration is one such step in the overall treatment process of water and waste water. Although the research done in the past three decades has provided a better understanding of the filtration process, the mechanisms involved and the variables affecting the process, the applications of the process have not radically changed. Filtration, probably, is still the least understood process, yet it is the oldest of all.

Filtration as a Water Treatment Process

Filtration may be defined as the separation of suspended solids from a liquid by passing it through a filter media, such as diatomite, perlite, sand, gravel or anthracite, using a pressure difference as the driving force. The entire bed thickness of the filter media might be effective in entrapping the solids, as in a rapid sand filter, or only the top shallow layer might be effective, as in a slow sand filter or even a precoat diatomite filter. The filtration in the first filter, therefore, is a case of <u>depth</u> filtration and that in the second situation, is a case of <u>surface</u> filtration or <u>cake</u> filtration. The designation of any process as either depth filtration or surface filtration, <u>completely</u>, is an

oversimplification. The classification is valid only in a relative sense. Both depth and surface removal occur in all filters.

In depth filtration through a porous media, the suspended solids may be removed by straining or by adsorption. Straining takes place when the particle removed is larger than the filter pores or interstices through which the water has to pass. Adsorption of these particles and of particles too small to be removed by straining takes place when the particles come close enough to the wall of the filter media to be retained there by the adsorptive forces at work on both the filter media and the suspended particles. In order to be removed by adsorption, the smallest suspended particles must be transported from the bulk of the flowing water over to the wall of the filter media where adsorptive forces can retain them. Thus, removal of suspended particles by adsorption are controlled by both transport and by adsorption phenomena.

Removal of suspended solids by filtration is thus the sum of the surface removal predominantly by straining and of the depth removal predominantly by straining and adsorption. Filters must be designed to remove the suspended particles which contribute most significantly to the pollution of the water. These particles may range in size from viruses (<<< 1 micron) to particles several orders of magnitude

greater in size (50-100 microns or larger). The larger particles may be removed efficiently with normal depths (24-30 inches) of coarse media such as sand and anthracite The smaller particles may be removed more efficiently with much small depths (0.1-0.5 inch) of much finer media such as perlite or diatomite, where the finer media serves to increase the filter media surface available for adsorption, to decrease the distances over which the particles must be transported to be adsorbed on the media surface, and to decrease the media pore sizes to enhance straining.

In general, it is not sound to expect one filter to be able to remove efficiently particles one or more orders of magnitude of difference in size. Thus, today we use deep, coarse media filters predominately in potable water filtration (together with chemical pretreatment to enhance particle growth and particle adsorption potential) to produce a water whose final turbidity is in the range of 0.5 to 5.0 units. In those situations where suspended particles are both low in concentration and small in size (< 1 to 20 microns), smaller media sizes and depths are used (diatomite filters). Today, we use only a single-stage filter (sand filter or diatomite filter) in potable water filtration practice. It is however, conceivable that we might ultimately improve filtered water quality by using a two-stage process in which coarse media are used to remove the largest

fraction of the suspended solids and fine media are used to polish the effluent. This study was directed at increasing our knowledge about the fine media or diatomite filters.

In current practice, diatomite or perlite filtration (hereafter referred to as precoat filtration) is a two-step process. The first step involves the laying of a thin protective layer of filter aid, generally diatomite or perlite as a "precoat" on a porous filter septum which supports the filter media throughout the filter run. This precoat build-up is accomplished by recycling a filter aid slurry (approximately 0.10 - 0.20 lb. of the filter media per sq. ft. of filter area) through the filter system. The purpose of the precoat is to protect the filter septum from getting fouled by the solids to be removed. It also acts as the initial filter medium. Precoat filtration is unique however, in that additional filter media (body feed) is added continuously in a suitable proportion to the raw water as it is delivered to the precoated filter septum. The addition of body feed prevents rapid blinding of the precoat and maintains a high filter cake porosity during the run. A complete filtration cycle includes 1) precoating the septum, 2) filtration of the raw water with a suitable proportion of body feed added and the formation of a cake containing the raw water impurities removed by the filtration and the body feed filter media on the precoated septum and

3) backwashing the filter and draining the precoat and the cake to waste.

Diatomite is composed of fossil-like skeletons of microscopic water plants called diatoms, ranging in dimensions from under five to over one hundred microns. Before it is ready for use as a filter aid, it is processed and the operations after quarrying include drying, crushing, calcination and air classification into various grades on the basis of particle size distribution and filtering quality. With its typical bulk density of about 15-20 lbs per ft^3 and a specific gravity of about 2.3 (3), diatomite would have a porosity ($\frac{\text{volume of voids}}{\text{total volume}}$) of about 80 to 90 percent. The major constituent of diatomite (about 90 percent) is silicon dioxide (SiO_2) (3).

Perlite is a material similar in outward appearance and hydraulic characteristics to diatomite. Perlite is a siliceous rock containing 3 to 5 percent water (4). When crushed and heated during processing, the rock expands to provide a mass of glass bubbles with many sealed cavities. Some typical characteristics of this material are (3,4):

Bulk density	9.5-13.5 lbs/ft^3
Specific gravity	2.0-2.3 lbs/ft^3 (average basis)
Silicon dioxide	73.2 percent

Thus, perlite may have about the same or even a little higher porosity than diatomite (80-90 percent) and similar

filtering characteristics. The properties of both these materials, diatomite and perlite, are discussed in detail elsewhere (3, 4, 5). Depauw (5) also discusses the processing of the diatomite in greater detail.

One of the most important criteria in any engineering operation is economy. In precoat filtration, this would mean design of a filter to produce a desired quality of the filtered water at least cost. Baumann and LaFrenz (6) observed that filtration cost is a function of a number of variables, the most important of them being the cost of filter aid, labor, power, and the fixed cost of the plant itself. Dillingham (7) investigated the problem of optimization of the precoat filtration plant to a great depth and wrote a computer program POPO[1] to achieve that objective. A POPO-like program can be used to determine the optimum combination of unit flow rate (q, in gallons per minute per sq ft of filter area), body feed concentration (C_D, in mg/l), and terminal head loss (H, in feet of water) that will result in production of filtered water at minimum cost using that grade of filter aid used as precoat and body feed. Since the <u>total</u> filtration cost stems, essentially, from the cost of the filter aid used, the labor cost, the cost of the power required to run the pumps, and the cost of the plant itself

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[1]POPO means "Program for Optimization of Plant Operation."

(6), the input to POPO, or any other program designed for a similar purpose, will be such so as to enable the computer to calculate these itemized costs. Even manual calculations are possible, although admittedly, optimization of all the factors such as flow rate, body feed concentration, terminal head loss and selection of a particular filter aid out of several available, is a more laborious and a difficult job without the aid of a high speed computer.

The computations of some of the component costs such as the labor, the power and the fixed plant cost are relatively easy and require a knowledge of only the unit cost of these components. The most important of these costs is the cost of the filter aid itself, the determination of which is rather difficult. Besides the unit cost of the filter aid (cents per lb), a method is needed to determine the length of a filtration run for a particular flow rate and body feed (C_D) in filtering a particular water to reach a predetermined head loss. In other words, the engineer confronting the optimization problem, must be able to predict with a reasonable accuracy, the course of any filtration run (head loss versus filtration time) for the water he plans to filter choosing a set of filtration variables. It is conceivable, however, that the optimum combination of the design variables in one situation, may not be optimum, in another situation because of the different unit costs and the

different characteristics of the raw water. This optimum combination will even be different at different times at one particular plant location, if the raw water quality is continuously fluctuating. A full-scale pilot plant has been used to collect the data necessary for precoat filter plant design. However, the process of design would be greatly simplified if laboratory testing could be used instead of field pilot-plant testing to collect the data required for plant design.

The purpose of this thesis was to evaluate all possible techniques which might be used in the laboratory for collecting the data necessary for the optimization of precoat filters for water filtration. Of major importance was the development of a simple laboratory apparatus which could be used to predict the filter operationg characteristics using a limited quantity of the raw water and the filter aid. The specific thesis objectives will be discussed in a subsequent chapter.

LITERATURE REVIEW

General

Before attempting to solve any problem, the study of the problem in the greatest possible detail is extremely important. In the context of filtration, this would mean a detailed study of the mechanisms involved in this process and the variables affecting each mechanism. Burns (8) presented an excellent discussion of these mechanisms and the variables affecting them. Several researchers have contributed in the past to the understanding of the filtration process and such contributions are still continuing. The filtration process is so complicated that it looks impossible that a design engineer will ever have a mathematical formula at his disposal that would be applicable in all situations. Attempts to formulate such a model (7, 9, 10, 11, 12, 13) have been made in the past although the practical applicability of the mathematical models presented by these engineers is seriously questionable. A full appreciation of the above statement requires a study of the mechanisms involved in this complex process called filtration.

Mechanisms Involved in Filtration

Suspended particle removal mechanisms can be subdivided broadly into two groups: physical removal mechanisms and

chemical removal mechanisms.

Physical removal mechanisms

The mechanisms included in this category are:

1) direct sieving or straining
2) sedimentation
3) inertial impingement and centrifugal collection
4) Brownian movement
5) chance contact
6) diffusion.

These mechanisms depend on several physical and filter operational variables (14), some of which are:

1) filter media size and depth
2) suspended particle size
3) suspended particle density
4) filtration rate or fluid velocity
5) fluid viscosity
6) fluid density
7) temperature.

The list of the physical removal mechanisms and the list of variables affecting these are not complete. However, the discussion of these will provide a good background to an understanding of the physical aspect of the filtration process.

Straining A particle is likely to be removed by straining when its size is larger than the size of the opening or the size of the pore in a filter bed. Traditional concepts have considered straining as the primary removal mechanism in filtration. It can easily be shown that spherical particles bigger than seven percent of the size of the spherical particles constituting the filter media can be removed by straining alone. This, however, assumes that the suspended particles in the raw water and the filter media are both spherical in shape. Such a general conclusion is not possible in an actual filtration situation, when this assumption may be valid only by mere chance. Since most suspended solids and filter media will not be spherical, an observation that is especially true for diatomite filter media, it is probable that the particles which are completely removed by straining alone may even be smaller than 7 percent of the filter media size. In the case of diatomite filtration, direct removal by straining through the pores of a diatom is also possible.

It is evident, therefore, that smaller filter media particles (which implies smaller pore openings) and larger suspended particles will enhance the chances of removal by this mechanism. O'Melia and Crapps (14) state that the probability of removal, p_s, of a suspended particle of diameter, D, using sand with diameter, d, as a filter media

is equal to $(D/d)^{3/2}$.

Sedimentation According to the advocates who claim that sedimentation is one of the possible removal mechanisms in a filter (15, 16), the void spaces in a filter, particularly a rapid sand filter, act as settling basins. Thus, all the variables affecting sedimentation such as fluid density, particle size, gravitational constant, particle density and fluid viscosity, according to this postulation, would affect suspended solids removal. Ives (9) and others have postulated that the gravitational forces remove suspended solids from the flow stream (transport step), and that adsorptive forces contribute to the retention of a suspended particle when it reaches the surface of a sand grain (attachment step). Sedimentation may be of importance where the suspended solid particles are rather large and have a specific gravity much higher than that of the fluid. Sedimentation will, according to Stoke's Law, not be effective when this is not true. It is conceivable that this mechanism would play little part in the removal of colloidal particles with densities close to that of the liquid.

Inertial impingement and centrifugal collection This is predominately a transport mechanism. The motion of a particle through a filter bed is essentially haphazard and random. Whenever the tangential velocity component of any

suspended particle intercepts a filter media particle, the particle will move towards the filter media and possibly be removed by impingement. Continually changing stream lines caused by the random orientation of the void spaces may result in removal by this mechanism.

Due to the curvilinear arrangement of the pores, the suspended particle will also be constantly subjected to a centrifugal force tending to throw the particle away from the center of the pore. This may also cause impingement and, thereby, the removal of the particle. Inertial impingement is reported to be a predominant removal mechanism in fibrous filters used to cleanse air (14).

Brownian movement Brownian movement, a transport mechanism, is indicated by the to and fro darting of the suspended particles and is caused by the collisions of the fluid particles with the suspended particles. For a one-micron spherical particle suspended in water, the translation due to Brownian movement might be about 0.3 micron (8). The suspended solid particles can be brought into contact with the surface of the filter medium as a result of this translatory movement and thus be removed. Brownian movement will be much more effective in the transport of particles less than 1 micron in size and ineffective in affecting transport of larger particles.

Chance contact Chance contact of the suspended solid particles with the filter medium particles can result as a consequence of the converging fluid stream lines in a filter bed. O'Melia and Crapps (14) state that the probability of removal of a suspended particle by this mechanism is proportional to the square of the diameter of the suspended particle and inversely proportional to the third power of the filter media particles, $\frac{D^2}{d^3}$. Thus, the removal probability by this mechanism will increase with increasing suspended particle diameter and decreasing size of the filter media particles.

Diffusion Friedlander (17) proposed that diffusion is a primary transport mechanism in filtration. His observation was based on aerosol filtration, but it by no means implies that this is not important in water filtration. O'Melia and Stumm (18) calculated contact efficiencies for a typical rapid sand filter as a function of the diameter of the particles in suspension, based on Friedlander's diffusion model, and found that the results of these calculations were not in disagreement with experimental observations in water filtration practice. This might indicate the possibility that diffusion may be an important transport step in water filtration as well. The above authors also suggested that diffusion would have minimal transport effects on particles in their example, 3 microns in size or larger. Diffusion

would be more effective in transport of suspended solids for suspended particles smaller than some critical size.

Chemical removal mechanisms

The physical removal mechanisms hypothesized above have failed to explain some of the observed occurrences, both in laboratory and field filter performance. For example, Burns (8) filtered clay and organic suspensions (Folger's coffee) through diatomite cakes and found that these solids were removed when the diatomite was coated with a cationic polyelectrolyte. Without such a coating, no removal was effected. O'Melia and Crapps (14) studied the effect of the presence of anions (chlorides, phosphates and sulfates) in filtering iron bearing waters through a bed of sand. They observed that phosphates cause a deep floc penetration in the sand bed, making longer filtration runs possible. A similar but lesser effect was noticed when chloride and sulfates were used as anions. Cations were not at all effective. Many more observations are hard to explain on the basis of the physical removal mechanisms. Thus, some chemical mechanisms important in the removal of suspended solids during filtration have also been proposed from time to time.

Van der Waals forces These are molecular cohesive forces between particles, increasing in intensity as the

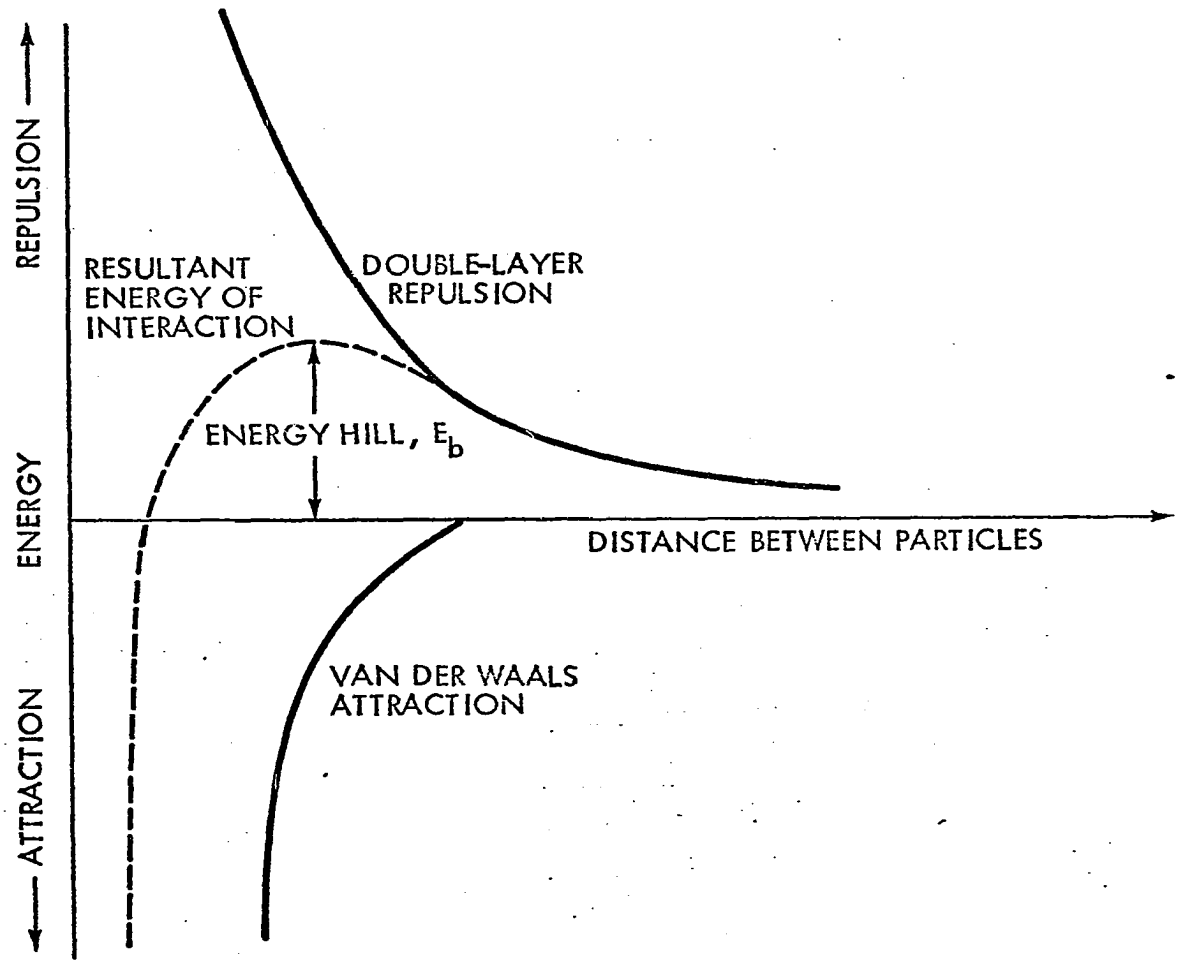
particles approach each other. Van der Waals forces are important in physical adsorption.

Day and Selbin (19) discuss three different sources of Van der Waals forces which are 1) dipole-dipole interaction between polar molecules such as those of water, 2) dipole induced-dipole interactions resulting from the polarization of one molecule by the dipoles of the surrounding molecules, and 3) London dispersion forces resulting from the interaction of electron shells.

The magnitude of these attractive forces between atoms is inversely proportional to the seventh power of the separation distance. For suspended particles, however, these attractive forces vary according to the inverse square rule (14). When the two colloidal particles (or suspended particle and a filter media particle, for example) get extremely close to each other by agitation, mixing, Brownian movement or by any other transport mechanism, these attractive forces play a significant role in causing flocculation and eventual removal of the floc thus formed or attachment of the suspended solid to the filter media. See Figure 1 (20).

Electrokinetic forces Oulman and Baumann (21) and Burns (8) reported that the removal of clay was enhanced through a diatomite cake, when the charge on the diatomite

Figure 1. Energy of interaction between two colloidal particles in the form of attraction by Van der Waals forces and repulsion by electrostatic forces


REPULSION
ENERGY
ATTRACTION
RESULTANT ENERGY OF INTERACTION
DOUBLE-LAYER REPULSION
ENERGY HILL, E_b
DISTANCE BETWEEN PARTICLES
VAN DER WAALS ATTRACTION

was reversed from negative to positive by the use of coatings with iron or aluminum salts or with cationic polyelectrolytes. Burns (8) also found that this charge reversal was pH dependent, and a positive charge existed with the coating he used only in the acidic range of pH. These and many other phenomenon can be explained if the electrical nature of the particle surfaces is recognized. The reasons for the development of a charge on the surface of the solid particles, when placed in contact with water, may be one or more of the following (14):

1) ionization of molecules at the particle surface
2) unsatisfied charges because of imperfections in the crystal lattice
3) chemisorption with specific ions in solution
4) weaker, physical adsorption of ions from solution, as produced by hydrogen bonding or Van der Waals forces.

Several theories explaining the complete cross section of the solid-liquid interface in such situations have been proposed, namely, the Helmholtz Electric Double Layer Theory, the Gouy Double Layer Theory and the Stern Double Layer Theory (22). Without discussing the merits and demerits of these proposed theories, which might not be very relevant in the present context, the common points of all the theories are cited here. At the solid-liquid interface, a tightly held

layer of ions of opposite charge (i.e., cations in the case of clay or diatomite, both of which acquire negative surface charge) termed the "stationary layer" and a second, more loosely bound layer of ions, also of opposite charge, termed the "diffuse layer" are produced. This double layer always exerts a repulsive potential between similar particles in an aqueous phase. Literature provides enough evidence to indicate that the filtration process can be dependent upon these surface characteristics of the filter media and the suspended particles. These surface characteristics will be affected by the ions in the fluid. It is also conceivable that by a charge reversal on the surface, the filtration characteristics can be modified and improved as demonstrated by Burns (8) and Oulman and Baumann (21).

Fair et al. (20) point out that the repulsive energy caused by the interaction of these double layers on similar particles decreases more or less exponentially with distance. Figure 1 shows both the Van der Waals attractive energy and the double layer repulsive energy between the filter media and a suspended particle or between two colloidal particles in water. The figure also shows the resultant energy. It is evident that when the particles are very close to each other (less than a few hundred angstrom units), the resultant is an attractive energy. However, to approach each other, the particles must surmount the energy hill, E_b. The kinetic

energy of the particles must be large enough to enable them to cross this energy barrier.

The mechanisms described in the foregoing pages are far from complete. The purpose of explaining some of these mechanisms was only to describe the complexity of the filtration process to a cursory student of filtration, who may be apt to look upon this process as simple and unchallenging. As this thesis progresses, more and more evidence might be presented in support of this. The above presentation is not concerned with the applicability of these phenomenon to any specific filtration system.

Filtration Processes

The filtration processes can be broadly classified in the following categories:

1) Constant pressure filtration: In constant pressure filtration, the pumping head across the filter media is constant throughout the run. Due to the build up of the cake of suspended solids and body feed, or of the suspended solids alone if no body feed is used, the flow rate through the filter continues to drop during the filtration run as the resistance of the filter media and its load of suspended solids increases. The run is terminated as soon as a predetermined low flow rate is reached. At the beginning of the run, the entire pressure drop, which is equal to the pumping

head, is across the septum or the precoated septum, whatever the case may be. As the filtration run progresses and the cake builds up, the pressure drop across the septum alone keeps on decreasing. The pressure drop across the cake, however, increases by an equal amount so that the total pressure drop always remains the same.

2) Constant rate filtration: In constant rate filtration, the filtration is carried out at a constant flow rate. Due to the build up of the cake during the run, the pumping head has to be increased continuously to cope with the increasing resistance, as flow rate = $\frac{\text{driving force}}{\text{total resistance}}$.

As soon as the terminal pumping head is reached, the run is terminated. It is difficult to say what proportion of the increased headloss in a particular time interval will be through the septum and through the cake. However, it has generally been assumed that the entire headloss increment would be through the cake. This assumption implies a constant resistance of the septum and its precoat (23).

3) Constant rate filtration followed by constant pressure filtration: It is possible, although rarely practiced, to filter at a constant rate until the terminal pumping head is reached and then to continue the filtration at that constant head. This combined process is impractical and seldom employed. Ruth (24) pointed out, however, that this process can accomplish a required amount of filtration

within a given pressure limit in less time than either method alone.

Other methods include variable rate-variable pressure filtration and stepped pressure filtration, but these are not of much interest where this thesis is concerned. Only the first two processes are of interest.

Precoat Filtration (Constant Rate)

Precoat filter operation

The precoat filtration process was explained briefly in Chapter I. It was mentioned that a complete filtration cycle consists of the three distinct operations discussed below.

Precoating In the first operation, a thin layer of precoat (about 1/8 inch-1/10 inch thick) of the selected filter aid (approximately 0.10-0.20 lb of diatomite or perlite per sq ft of filter area) is deposited on a porous filter septum, which supports the filter media throughout the filter run. This is accomplished by recycling a filter aid slurry containing the calculated weight of the precoat filter aid suspended in clean water through the filter until nearly all of the filter aid is deposited on the septum. The precoat serves not only to protect the filter septum from getting fouled by the suspended solids in the raw water,

but also acts as the initial filter medium. The precoating operation is usually accomplished at a flow rate equal to or greater than that used in the subsequent filtration cycle.

Filtration In the second operation, additional filter aid (body feed) is added to the raw water in a suitable proportion to the suspended solids present before the raw water is delivered to the precoated filter septum. The addition of the body feed prevents blinding of the precoat permitting longer filtration runs. There are several grades of the filter aids (both diatomite and perlite) available commercially based on the media particle size distribution. Coarser grades are more expensive than the finer grades but allow longer filtration runs for the same terminal head loss. Fine grades are relatively less expensive but form a relatively impervious cake and thus cause a faster head loss buildup. The effluent produced by the use of coarse grades may be inferior to that produced when the fine grades are used as a filter aid. This is due to the bigger void openings in the coarse grades of the filter aid.

The effect of the body feed concentration also deserves comment. If too little body feed is fed, head loss will increase exponentially with increasing time of filtration indicating a compressible cake. Increasing body feed dosages will tend to flatten the head loss vs time curve until an optimum concentration of the body feed will produce a linear

head loss vs time curve through a flat filter septa indicating a nearly incompressible cake. The use of the body feed in proportions greater than the optimum may be uneconomical, as any benefit resulting from the excess amount in terms of an increased porosity could be more than offset by the additional thickness of the cake. This has been demonstrated in a number of studies including those by Carman (25) and LaFrenz (26).

The question of choosing the optimum grade and concentration (C_D) of the filter aid to be used as body feed is consequently a difficult one. In industrial filtrations using existing equipment and batch instead of continuous filter operations, the question may revolve around finding the optimum grade and concentration of filter aid to use to minimize only the filter aid costs. In the design of a new filtration plant for continuous operation, the answer lies in selecting a filter aid that produces the desired quality of the filtered water at least cost. In fact, the overall economy can be achieved by an optimum combination of several factors, such as grade and type of the filter aid, body feed concentration, terminal head loss and flow rate. This has been discussed in greater depth in the first chapter.

Backwashing After the terminal head loss across the filter septum, precoat, and cake is reached the third operation begins. The filtration cycle is interrupted and the filter is backwashed using clean water. The precoat and

cake thus washed from the septum are drained to waste.

In potable water cake filtration, the constant rate filtration process is adopted most frequently.

Theory

At the end of the precoating operation, the filter housing is usually full of the clean water used in suspending the precoat filter media during precoating. During the filtration cycle the raw water gradually replaces this clean water. It takes some time, depending upon the flow rate and the volume of the system including the housing before the clean water is completely replaced at the start of the filtration cycle. As a result the head loss development during this period, known as the initial dilution period, is much less than it is after the housing is completely full of the raw water to be filtered.

Furthermore, when filtering water through cylindrical septa, the increasing cake thickness continuously increases the outer surface area of the filter cake. This causes the flow rate per unit area of the cake to diminish as the run progresses and the surface area increases. The decrease in flow rate per unit area which thus results during the run causes the rate of head loss increase to diminish continuously during the run. This effect is known as an increasing area effect. It follows, therefore, that the increasing area

effect will be more predominant using cylindrical septa with smaller diameter than with larger diameter septa for the same conditions of the filtration run. This effect would be absent in systems using flat septa.

Based on D'Arcy's law (which assumes laminar flow), Dillingham et al. (23) developed filtration equations, which included the above two effects: the initial dilution effect and the increasing area effect for cylindrical septa. The earlier attempts (26, 27) in the development of such equations ignored both of these effects. Another merit of the equations proposed by Dillingham et al. (23) lies in the fact that they took viscosity into account as a separate term and, therefore, the specific resistance (β-index) was independent of water temperature. This was not done in earlier attempts. The equations developed by Dillingham et al. are summarized in Figure 2.

The increasing area effect, as explained above, can be quite significant. Baumann et al. (28) showed that a body feed concentration of 76 mg/l of Sil-Flo 272 (a coarse grade of a typical perlite) in a flat septa situation would be required to provide the same performance (100 ft. water head loss in 6 hours) in filtering a synthetic Fe^{+++} bearing water as a body feed concentration of 59 mg/l using 3-1/2 inch O.D. cylindrical septa under exactly identical conditions (C_s = 4.0 mg/l, q = 1 gpm/ft^2, temperature = 68° F,

Figure 2. Summary of precoat filtration equations

	Equation	
Any septum	$H_p = q\nu\xi w/g$	(1)
Cylindrical septum	$H_c = (R_S\sigma/\phi)\ln(1 + R_S\ \phi\ X/R_o^2)$	(2)
	$L = \sqrt{R_o^2 + R_S\ \phi\ X} - R_S$	(3)
Flat septum[a]	$H_c = \sigma X$	(4)
	$L = L_p + \frac{\phi X}{2}$	(5)
where,	$\delta = Q/V_f$	
	$\sigma = q^2\nu\beta C_D/g$	
	$R_o = R_S + L_p$	
	$\phi = 2q\gamma_w C_D(10)^{-6}/\gamma_p$	
	$L_p = w/\gamma_p$	
	$X = t - (1 - e^{-\delta t})/\delta$	

Symbol	Meaning	Dimension
A_s	Septum area	$[L^2]$
β	Filter cake resistance index or β-index	$[L^{-2}]$
C_D	Body feed concentration, ppm by weight	$[-]$[b]
C_S	Suspended solids concentration, ppm by weight	$[-]$[b]
δ[c]	Dilution rate, theoretically Q/V_f	$[T^{-1}]$

[a] Septum that does not exhibit increasing area effect

[b] Dimensionless

[c] δ will be equal to Q/V_f in a completely-mixed system in a filter housing. Since the conditions in practice are far from ideal, values of δ should be estimated by visual selection of the inflection points on the head loss versus time curve in any particular filter run with the assumption that at this time δt is approximately equal to 3 (7).

Symbol	Meaning	Dimension
g	Gravity constant	$[LT^{-2}]$
γ_p	Bulk density of precoat	$[FL^{-3}]$
γ_c	Bulk density of filter cake	$[FL^{-3}]$
γ_w	Density of water	$[FL^{-3}]$
H_p	Headloss through precoat	[L]
H_c	Headloss through filter cake	[L]
L_p	Thickness of precoat	[L]
L_c	Thickness of filter cake	[L]
L	$L_p + L_c$	[L]
ν	Kinematic viscosity of influent	$[L^2T^{-1}]$
Q	Flow rate	$[L^3T^{-1}]$
q	Flow rate per unit septum area	$[LT^{-1}]$
R_S	Outer radius of septum	[L]
R_o	Outer radius of precoated septum = $R_S + L_p$	[L]
t	Time since start of filtering cycle of a particular run	[T]
V_f	Volume of housing	$[L^3]$
w	Precoat weight per unit area	$[FL^{-2}]$
X	$t - (1 - e^{-\delta t})/\delta$	[T]
ξ	Filter aid resistance index or ξ index	$[F^{-1}L]$

γ_c = 10.61 lb/ft^3). This represents a saving of 22 percent of the body feed filter aid by using cylindrical instead of flat septa. The use of the cylindrical septa may, therefore, be quite economical in certain cases.

The following assumptions were made in deriving the equations in Figure 2.

1) Q remains constant (constant-rate filtration).
2) The body feed rate is sufficient to form an incompressible cake.
3) The in-place bulk densities of the precoat (γ_p) and filter cake (γ_c) remain constant.
4) The concentration of solids being filtered (C_s) and the body feed concentration (C_D) remain constant.
5) No solids pass through the cake.
6) No concentration gradients exist in the filter housing (as in a completely-mixed system).
7) The suspended solids from the raw water which are retained in the filter cake do not increase the cake thickness appreciably.

Assumptions 1, 3, and 4 are reasonably valid during a filter run. Assumption 2 will be the least valid in a filter run with no body feed and will be the most valid with a lot of body feed. In a practical situation where enough body feed would be used to form the incompressible cake which is essential for optimum performance, assumption 2 might be con-

sidered valid. Assumption 5 is also true since the filter cake should not allow a significant fraction of the suspended solids to pass through. Assumption 6 may not always be true in a practical situation in at least the early part of the run, since complete-mix conditions may not be present within the filter housing. However, if the values of the dilution factor, δ, are read from the head loss versus time curve as explained in a footnote in Figure 2, the assumption may be considered valid without affecting the results seriously. The last assumption may also be considered true, as the concentration of suspended solids (C_s) is generally far less than the concentration of body feed (C_D). This assumption will not be valid when C_s exceeds C_D (C_s/C_D greater than 1 as in a compressible cake).

The β-index in the equations in Figure 2 is a measure of filter cake resistance and is known as specific filter cake resistance or simply the specific resistance. Knowing the β-index for the conditions of a particular run, the head loss versus time curve can be predicted for that run. Thus, the ability to predict values of this index under a variety of conditions is extremely important if we are going to predict the head loss vs time of filtration relationship for an intended run in a practical situation. The β-index depends on a number of factors such as the nature of the suspended solids, their concentration (C_s) in the raw water,

the filter aid characteristics, and concentration of body feed (C_D). Since all of these are constant during a filter run, the β-index will remain constant during the run. However, this index will vary from one run to the next as the raw water quality changes or as changes are made in C_D.

Dillingham et al. (29) developed several β-index prediction equations which are summarized below.

$$\beta = 10^{b_1} (C_s/C_D)^{b_2} \cdot (C_D)^{b_3} \cdot \xi^{b_4} \quad (6)$$

$$\beta = 10^{b_1} (C_s/C_D)^{b_2} \cdot (C_D)^{b_3} \quad (7)$$

$$\beta = 10^{b_1} (C_s/C_D)^{b_2} \quad (8)$$

where

b_1, b_2, b_3 and b_4 are exponents determined empirically from the results of several runs made on one particular water with varying C_s/C_D from one run to the other.

Use of the additional variables in Equation 6 and 7 (as compared to Equation 8) improved cake resistance prediction in the case of some suspended solids (7). If β-index were only a function of C_s/C_D, using the same water and the same filter aid, use of Equation 7 should not improve the prediction over Equation 8 under any circumstances. Either Equation 7 or 8 can represent the results of a group of filter runs using the same grade of filter aid ($b_4 = 0$). Equation 8 suggests that log β plotted versus log C_s/C_D

would yield a straight line. The validity of these equations, including those in Figure 2, has been checked in several studies at Iowa State University (3, 30, 31, 32, 33).

Sutherland (34) hypothesized that cake filtration consists of a series of blocking filtration steps. He reasoned that any infinitesimal layer of the cake is like a septum to the layer above it and serves to block this septum. Using this argument, he developed a relationship according to which C_s/C_D when plotted against log specific resistance (β-index) would yield a straight line. The author presented some experimental data which conformed very nicely to the above relationship. The disagreement with the equation proposed by Dillingham et al. (Equations 8), which indicates a straight line relationship only between log C_s/C_D versus log β, was attributed by the author to the fact that these investigators kept the suspended solid concentration (C_s) constant in most of their work and that they varied C_s/C_D by varying the body feed concentration only. This observation, however, does not appear to be correct in some cases (32, 33).

Oulman and Baumann (35) suggested that C_s/C_D is not a good parameter to use. They proposed that C_s/V_v is a more fundamental variable (where V_v is volume of voids in the clean cake) and when plotted against β-index on a log plot (i.e., plot of log C_s/V_v versus log β) would give one single

straight line for any particular water irrespective of the filter aid used. In other words, the important variable according to this hypothesis, is the fraction of the voids that have been filled with the solids removed. The specific resistance of the cake, therefore, will be constant for a particular value of C_s/V_v for any particular water. The authors came to this conclusion on the basis of analysis of limited data (28) collected on Fe^{+++} bearing water only, where C_s was approximately equal to 8.0 mg/l.

According to this conclusion then, the β-index would always be the same on a particular water so long as C_s/V_v remains the same. In other words, two filtration runs made, for example, in an Fe^{+++} bearing water with C_s = 2 mg/l, C_D = 25 mg/l of a particular filter aid and C_s = 4 mg/l, C_D = 50 mg/l of the same filter aid would yield the same value of β-index under identical filtration conditions, since both C_s and V_v have increased twofold in the second run. This means, then, that the head loss would be exactly twice in the second run using flat septa, since head loss = βC_D. (See Equation 4 in Figure 2).

It was mentioned earlier that the β prediction attempt improved considerably with the use of Equation 7 as compared to Equation 8 on some waters (7). This observation also negates, to some extent, the above reasoning of Oulman and Baumann (35). Regunathan (32) observed a decreasing value

of specific cake resistance, β-index, with increasing concentration of sodium montmorillonite solids using Hyflo Supercel as a filter aid in his runs conducted at the same value of C_S/C_D. This observation cannot be explained on the basis of the hypothesis of Oulman and Baumann (35).

The decreasing specific resistance with increasing solids concentration has been reported in a series of articles in the literature (24, 36, 37, 38 and 39) although the reasons cited for these observations were not the same. All these observations, however, were made on the basis of results collected in constant pressure filtrations. Except for the brief data mentioned above (32), no other data seems to be available from studies conducted with a constant rate of filtration. Ruth (36) attributed this phenomenon of decreasing specific resistance with increasing solids concentration to electroosmosis. He said:

> If the same hydrostatic pressure difference is employed to force liquid through long and short capillary tubes of the same diameter (that is, thick and thin filter cakes), the greater volume rate of flow through the shorter capillaries will generate a correspondingly larger streaming potential gradient and diaphragm current. Since an increase in current flow corresponds to a higher electroendosmotic pressure intensity opposing the applied hydrostatic pressure difference, the ratio of volume rate of flow through the two lengths of capillaries will in general be less than the expected ratio -- that is, the inverse ratio of their lengths. Hence, when the data is interpreted on the basis of resistance to fluid flow per unit length (or weight of solids), the specific

resistance appears to increase with decreasing capillary length.

Ruth's remarks seem to imply that this phenomenon of concentration dependence (i.e., dependence of specific resistance on the concentration of solids) would not occur in constant rate filtration, if the filtration rate is kept constant from one run to the next. This, however, does not agree with some of the earlier stated observations (32).

Tiller and Shirato (37, 38) gave a different reason for this concentration dependence. They felt that a thick cake resulting from a highly concentrated slurry will have a decreased average pore velocity (ft^3/ft^2 sec) when compared to the average pore velocity in a thin cake. To account for this, they introduced a "J factor" and recommended that the specific resistance should be multiplied by this factor. This new specific resistance was found to decrease with increasing slurry concentration in accord with the experimental data. They developed a cumbersome equation to calculate the value of this factor. Only with slurries containing 30-40 percent solids would the use of J factor affect the specific resistance significantly. For dilute suspensions containing 200-300 mg/l of solids (frequently encountered in potable water filtration practice) including the filter aid, J will be equal to one approximately. It may imply, therefore, that for such waters the specific resistance will be

independent of the solids concentration for all practical purposes, so long as the ratio of suspended solids concentration to body feed concentration (C_s/C_D) remains the same. The concentration dependence observed in some cases (32) does not, therefore, find any justification on this basis alone. Tiller and Shirato, however, derived their theory on the basis of constant pressure filtration. This does not seem to invalidate the above conclusion.

This phenomenon has also been explained (39) on the basis of the orientation of the cake particles as well. It was argued that the particles in a dilute slurry are more likely to orient themselves in the cake in a denser packing than the particles in a concentrated slurry where the freedom of movement (and thereby opportunity to pack densely) would be much less. Obviously, it is difficult to quantify such factors and no attempts were ever made in this direction by the protagonists of this theory except through qualitative reasoning.

These observations do pose problems which are beyond solution by the theories of Dillingham et al. (23, 29) and Oulman and Baumann (35).

Bishop et al. (40), as early as 1931, demonstrated conclusively the existence of electro-osmotic pressure in filtration. They found that when the pressure producing flow of a dilute potassium chloride solution through a

cellophane membrane was reduced to zero, a backward flow of fluid took place for a short time. They attributed this to the fact that the electro-osmotic pressure, which had developed before the pressure was reduced to zero, continues to exist for some time like the induced electromotive force in a capacitance electric circuit. During this short time, then, this electro-osmotic pressure would cause a backward flow of the fluid even though the hydraulic driving force was zero during that brief period. It is also evident that if the pressure were applied again before the osmotic pressure completely disappeared, the resistance upon resuming flow would be less than it had been when flow was stopped. Such behavior was actually observed by Bishop et al. (40). Similarly, if the ionic strength of the fluid were suddenly increased in the middle of the flow, the flow would be expected to increase because of a collapse of the double electrical layer from its original thickness to the smaller value corresponding to a greater ionic strength. The smaller thickness of the double electrical layer produces a smaller value of the osmotic pressure. Such a phenomenon was actually observed (40) when thorium ions were added to the above solution. The flow increased almost ten times. For the same reason it would be expected that the extent to which membranes are blocked (i.e., flow is retarded by the generation of osmotic pressure) by the passage of fluid

should vary with the ionic strength of the solution passed through. The same workers cite tests in which the resistance of a membrane to the passage of a univalent salt solution was observed to increase many fold, whereas little or no resistance increase was observed when an equal volume of a thorium salt of the same molar concentration was passed through a similar membrane.

This mechanism provides a simple qualitative explanation for another phenomenon often reported in literature. In several studies (41, 42, 43) it was observed that continued passage of demineralized water through a cake or any type of fine-pored membrane was accompanied by a gradual increase in resistance. Only after a lapse of a few hours, 15 to 24 hours in some cases, did the resistance become constant (36). This observation may have a reasonable explanation in the fact that growth of the electro-osmotic flow region proceeds from a thickness of zero to an equilibrium value, at a rate depending upon the extent to which the rate of mechanical transport of the fluid caused by hydrostatic pressure exceeds that of electro-osmotic transport of the fluid which is caused by electro-osmotic pressure. If these rates are nearly equal, a fairly long time might elapse before an equilibrium potential would be reached. The gradual increase in resistance simultaneously taking place is explained by the fact that as electro-osmotic velocity increases the absolute

velocity of flow is correspondingly decreased in a constant pressure filtration.

Abramson (41) showed that the retardation of flow due to electro-osmosis might be about 10 percent through a capillary 1 micron in radius. This retardation would increase for capillaries of smaller size.

Several other studies present strong evidence for believing that electrokinetic phenomenon play an important role in determining filtration behavior. Perhaps the effects of ions in solution on the filtration results reported by O'Melia and Crapps (14) have an electrokinetic basis. Unfortunately, it does not seem possible with the current knowledge to quantify the contribution due to this phenomenon to the overall filtration behavior such as flow rate and resistance. It, therefore, only adds to the long list of complexities and unknowns in filtration.

Filtration resistance defined by physical parameters of the cake The specific resistance term (β-index) defined by Dillingham et al. (29) does not provide a good concept of the physical nature of the cake. To a critical student of filtration, the specific resistance should correlate with the physical parameters of the cake such as its porosity, bulk density, the diameter of the cake particles, a shape factor and so forth. These characteristics are not only more fundamental but are easy to conceptualize. Such a student

is prone to think of a highly resistant cake in terms of a cake with a very low porosity consisting of particles of small diameter and so forth. He would, therefore, be interested in finding out how these factors affect quantitatively the specific resistance of the cake.

To be able to appreciate this comment a little better, two head loss equations, both for flat septa, will be derived from, essentially, the fundamentals in the following pages. No attention will be paid to the physical parameters in the development of the first equation. The second equation, however, will incorporate these factors.

Filtration equation with no regard to physical parameters The filtration of water, especially precoat filtration, is analogous to the flow of water through porous media. Thus, what is essentially D'Arcy's Law can be applied to filtration.

$$\frac{dV}{Adt} = \frac{dP}{\mu adL} \tag{9}$$

where

V = volume of filtrate $[L^3]$ filtered in time $t[T]$

A = gross cross-sectional area of porous media perpendicular to direction of flow $[L^2]$

$\frac{dP}{dL}$ = pressure gradient $[FL^{-3}]$

μ = dynamic or absolute viscosity $[FTL^{-2}]$

a = specific resistance $[L^{-2}]$

L = thickness of porous media (filter cake) in direction of flow [L]

or

$$q = \frac{\gamma_w dH}{\mu a dL} \quad \text{because} \quad dP = \gamma_w dH \tag{10}$$

where

γ_w = specific weight of the water flowing through $[FL^{-3}]$

or

$$q = \frac{\gamma_w i}{\mu a} \tag{11}$$

where

i = hydraulic gradient = $\frac{dH}{dL}$.

Now, kinematic viscosity $\nu = \frac{\mu}{\rho} = \frac{\mu}{\gamma_{w/g}} = \frac{\mu g}{\gamma_w}$.

Therefore,

$$\mu = \nu \gamma_{w/g}$$

Substituting this value of μ in Equation 11,

$$q = \frac{\gamma_w i}{\nu \gamma_{w/g} a} = \frac{ig}{a\nu}^{1} \tag{12}$$

$$= \frac{g\ dH/dL}{a\nu}$$

Therefore,

$$dH = \frac{qa\nu\ dL}{g} = q \frac{\nu}{g} \left(a \frac{V_c}{A}\right) \tag{13}$$

where

---

[1] In the usual form of D'Arcy's equation $q = Ki$; K, therefore = $g/a\nu$.

V_c = volume of the cake.

The factor in parenthesis in Equation 13 represents what is usually called cake resistance. The specific resistance "a" represents the resistance of a unit volume of filter cake per unit area and is, therefore, referred to as the specific resistance based on the volume of filter cake.

Since the thickness of the filter cake is difficult to measure accurately, it has been suggested that the specific resistance be referred to the weight of the filter cake, or preferably the weight of the filter aid in the cake, which is always known.

Therefore

$$dH = q\frac{\nu}{g} (Z_c \cdot W_D/A) \qquad (14)$$

where

Z_c = specific resistance based on the unit weight of diatomite in the cake per unit of area $[F^{-1} L]$.

W_D = weight of diatomite in the cake.

Applying Equation 14 to a particular instant during a constant rate filtration run, when the head loss is H^* and the weight of diatomite deposited in the cake through body feed is W_D: Since $W_D = Aq\ \gamma_w t\ C_D/10^6$, (15)

$$H = q\ \frac{\nu}{g}\ (Z_c\ W_D/A)$$

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*This assumes that head loss through the septum or precoated septum is negligible. This implies that the septum does not get significantly dirty during the run.

$$= q \frac{\nu}{g} Z_c (q \gamma_w t C_D / 10^6) \quad (16)$$

This equation should be valid after a filtration time of t using a body feed concentration, C_D, (mg/l of body feed).

By rearrangement of Equation 16,

$$Z_c \gamma_w / 10^6 = H g / q^2 \nu t C_D \quad (17)$$

or

$$H = q^2 \nu t Z_c C_D \gamma_w / g 10^6 \quad (18)$$

Therefore, if we let

$$Z_c \gamma_w / 10^6 = \beta [L^{-2}] \quad (19)$$

and substitute β into Equation 17,

$$\beta = H g / q^2 \nu t C_D \quad (20)$$

and

$$H = q^2 \nu t \beta C_D / g \quad (21)$$

Equation 21 applies to cake filtration through flat septa and was derived in almost the same manner by Dillingham et al. (23). (See Equation 4 in Figure 2).

Examination of Equation 21 reveals that for any filtration run at a constant rate q (ft^3/ft^2 sec) using C_D (mg/l) as the concentration of body feed, the head loss H in ft of water column can be found after any time interval t. The only unknown is β or Z_c, which has to be evaluated.

Filtration equation based on physical parameters

Several equations have been developed in the literature (13, 44, 45) to define specific resistance in terms of the physical characteristics of the cake. The following derivation is essentially due to Rich (44) based on the D'Arcy-Weisbach relationship, replacing conduit diameter D with an equivalent term 4 r_H.

$$H = \frac{f}{4} \frac{L}{r_H} \frac{\bar{V}^2}{2g} \tag{22}$$

where

H = head loss through the bed or cake, ft

f = friction factor, dimensionless

L = depth of bed or thickness of cake, ft

r_H = hydraulic radius of the pore in a filter cake, ft

$\bar{V}$ = average velocity of fluid through the pores of a filter cake, ft/sec

g = gravitational constant, ft/sec^2

The total pore volume of the filter cake can be considered equal to the volume occupied by the voids, or

$$\text{pore volume} = \frac{\varepsilon}{1-\varepsilon} N v_p \tag{23}$$

ε = porosity of bed = ratio of volume of voids to the total volume

N = number of particles in bed or cake

v_p = mean volume of a single particle, ft^3

The total wetted surface can be considered equal to the

total surface area of the solids, or

$$\text{wetted surface area} = N\, s_p \tag{24}$$

where

s_p = mean surface area of a single particle, ft^2

The hydraulic radius, $r_H = \frac{\text{pore volume}}{\text{wetted surface area}}$

$$= \frac{\varepsilon}{1-\varepsilon} \frac{v_p}{s_p} \tag{25}$$

or

$$r_H = \frac{\varepsilon}{1-\varepsilon} \frac{D_p}{6} \tag{26}$$

for a bed consisting of spherical particles of diameter D_p, ft. For non-spherical particles,

$$r_H = \frac{\varepsilon}{1-\varepsilon} \phi_s \frac{D_p}{6} \tag{27}$$

where

ϕ_s is a shape factor for the particles.

The value of ϕ_s for any filter media will depend on the particle shape characteristics of that media.

Also,

$$\bar{V} = q/\varepsilon \tag{28}$$

where

q is the superficial velocity through the filter cake in ft^3/ft^2 sec.

By combining Equations 22, 25, 27, and 28

$$H = f'(L/\phi_s D_p)(1-\varepsilon/\varepsilon^3)(\frac{q^2}{g}) \tag{29}$$

where

$f' = 3/4\ f$ = friction factor = dimensionless.

In the laminar flow region, the friction factor f' can be expressed as (44),

$$f' = K'(1-\varepsilon)/R_e \tag{30}$$

where

R_e = Reynold Number

or

$$f' = K'(1-\varepsilon)\mu/(\phi_s\ D_p\ q\rho) \tag{31}$$

where

K' = constant

ρ = mass of liquid in slugs/ft^3 = $\gamma_{w/g}$ $[ML^{-3}]$

μ = liquid viscosity $[FTL^{-2}]$

By substituting Equation 31 in Equation 29

$$H = K'(1-\varepsilon)\mu/(\phi_s\ D_p\ q\rho)\cdot(L/\phi_s\ D_p)(1-\varepsilon)/\varepsilon^3)\ q^2/g$$

$$= \frac{K'(1-\varepsilon)^2}{g\ \phi_s^2\ D_p^2\ \varepsilon^3}\ \frac{\mu Lq}{\rho} \tag{32}$$

or

$$\Delta p_{(lb/ft^2)} = \frac{K'(1-\varepsilon)^2}{\phi_s^2\ D_p^2\ \varepsilon^3}\ \mu Lq \tag{33}$$

$$= K'\left[\frac{(1-\varepsilon)^2}{36\ (v_p/s_p)^2\ \varepsilon^3}\right.\ \mu Lq \tag{34}$$

where

v_p = mean volume of a single particle, ft^3

s_p = mean surface area of a single particle, ft^2.

It is to be noted that the terms enclosed in the brackets are either constants or depend on physical characteristics of the cake. Since

$$q = \frac{1}{A}\frac{dV}{dt} \text{ at any instant} \tag{35}$$

Substituting this relationship in Equation 34 and combining terms,

$$\frac{dV}{dt} = KA\frac{\Delta p}{\mu L} \tag{36}$$

where

$$K = \frac{36\ (v_p/s_p)^2\ \varepsilon^3}{K'(1-\varepsilon)^2} = \text{permeability of the cake.} \tag{37}$$

Permeability is a property of the filter cake. Quantitatively, it is defined as the rate of flow of a liquid having unit viscosity across a cake of unit area and unit thickness when the pressure drop is unity. Its value depends upon such cake characteristics as porosity, average volume and surface area of the cake particles and so forth. Replacing K by 1/R where R is the cake specific resistance (lb force sec^2/lb mass ft^3),[1] Equation 36 becomes

$$\frac{dV}{dt} = \frac{A\Delta P}{\mu LR} \tag{38}$$

---

[1]Dimensionally equivalent to $[L^{-2}]$. R is the same as term "a" of Equation 9. Thus R and a have been used interchangeably.

where

$$R = \frac{K'(1-\varepsilon)^2}{36\ (v_p/s_p)^2\ \varepsilon^3} \tag{39}$$

The total filtration resistance is composed of the resistance of the cake plus the resistance of the filter septum (R_f). Therefore,

$$\frac{dV}{dt} = \frac{A\Delta P}{\mu(LR + R_f)} \quad . \tag{40}$$

Since

$$L = vV/A \tag{41}$$

where

v = volume of cake deposited per unit volume of filtrate which is approximately equal to the gross volume of clean diatomite in the cake per unit volume of filtrate. Then,

$$\frac{dV}{dt} = \frac{A^2\Delta P}{\mu(vVR+AR_f)} \tag{42}$$

It is difficult to calculate the thickness of the cake or the volume of the cake per unit volume of filtrate. A more convenient parameter to use is the weight of the diatomite in the cake per unit volume of filtrate (w). Then,

$$\frac{dV}{dt} = \frac{A^2\Delta P}{\mu(wVR'+AR_f)} \tag{43}$$

where

R' = specific resistance based on a unit weight of the diatomite in the cake.

Equating Equation 42 with 43,

$$vR = wR'$$

$$R' = \frac{vR}{w} = \frac{v}{w} \cdot \frac{K' (1-\varepsilon)^2}{36 (v_p/s_p)^2 \varepsilon^3} \quad . \tag{44}$$

Thus,

$$R' = \frac{v}{w} \cdot \frac{K' (1-\varepsilon)^2}{36 (v_p/s_p)^2 \varepsilon^3} = Z_c \tag{45}$$

Thus,

$$Z_c = \frac{K' (1-\varepsilon)^2}{36 (v_p/s_p)^2 \varepsilon^3} \frac{1}{\gamma_d} \tag{46}$$

where

γ_d = bulk density of the diatomite in the cake $[FL^{-3}]$.

Since

$$D_p = \frac{6}{\phi_s} \frac{v_p}{s_p}$$

$$Z_c = \frac{K' (1-\varepsilon)^2}{D_p^2 \phi_s^2 \varepsilon^3} \frac{1}{\gamma_d} \tag{47}$$

Equation 43,

$$\frac{dV}{dt} = \frac{A^2 \Delta P}{\mu (wVR' + AR_f)} \tag{43}$$

now integrates for a constant rate filtration to give

$$\frac{Pt}{V/A} = R'w\mu(V/A) + \mu R_f \qquad (48)$$

In other words, Equation 45 defines the specific resistance of the cake in terms of the physical parameters such as the bulk density of the cake, the average value of a single particle comprising the cake, the average surface area of a single particle and the average porosity of the cake. Knowing Z_c, β can be calculated from the relationship of Equation 19. As pointed out earlier, Equation 45 has a special merit since it provides a physical feeling about the specific filter cake resistance. How far this equation is useful to predict filter cake resistance in an actual practical situation will be discussed later at a more appropriate place.

Equation 48 can also be used instead of Equation 4 of Dillingham et al. (23) to calculate the specific resistance from the results of a filtration run made at a constant rate of filtration. In fact, this equation is not different than Equation 4 as shown below.

$$\frac{Pt}{V/A} = R'w\mu(V/A) + \mu R_f \qquad (48)$$

Neglecting the second term on the right hand side of the above equation, which represents the contribution to $\frac{Pt}{V/A}$ due to the septum, then

$$\frac{Pt}{V/A} = R'w\mu(V/A) \tag{49}$$

Now

$$P = \gamma_w H$$

$$w = \gamma_w C_D/10^6$$

$$\mu = \nu\rho = \nu\gamma_w/g$$

$$\frac{V}{A} = qt$$

Substituting these values in Equation 49,

$$\frac{\gamma_w Ht}{qt} = R' \gamma_w \frac{C_D}{10^6} \frac{\nu\gamma_w}{g} qt$$

$$\therefore H = R' \gamma_w \frac{C_D}{10^6} \frac{\nu\gamma_w qt\ q}{g\ \gamma_w}$$

$$= R' \frac{C_D}{10^6} \gamma_w \frac{\nu}{g} q^2 t \tag{50}$$

But

$$R' \frac{\gamma_w}{10^6} = \beta \qquad \text{(See Equations 19 and 45),}$$

or

$$H = \frac{\beta}{g} \nu q^2 t\ C_D = q^2\ \beta\ C_D\ \nu\ \frac{t}{g}$$

which is the same equation as Equation 4 developed by Dillingham et al. (23).

To use Equation 48 in calculating the specific resistance

from the results of a constant rate filtration run, plot $\frac{Pt}{V/A}$ versus $\frac{V}{A}$. For incompressible cakes, which is the assumption throughout all these developments, this plot would be a straight line. From the slope of the line, which should equal R' w μ, R' and then β can be calculated. See Appendix B.

Walker et al. (46) modified Equation 48 for compressible cakes so that

$$\frac{Pt}{V/A} = R'w\mu(\frac{V}{A} P^s) + \mu R_f \qquad (51)$$

where

s is the compressibility factor for the cake.

They suggested that Equation 51 be solved as a series of simultaneous equations, choosing sufficient experimental points to evaluate the constants.

Alternatively, they suggested s be determined from a few constant pressure runs on the same slurry at difference pressures. From a generally accepted relationship R' = R" P^s, s can be calculated, which would be the slope of log R' versus log P. The equations for the determination of R' for each of the constant pressure runs are basically the same as above, but will be explained in greater detail in the next section.

Thus, a plot of $\frac{V}{A} P^s$ versus $\frac{Pt}{V/A}$ will be a straight line for compressible cakes. From the slope of the line, which will equal $R'w\mu$, R' can be calculated from the results of a constant rate filtration run.

As mentioned earlier, several equations attempting to define the specific resistance in terms of the physical characteristics of the cake have been developed. Another such equation due to Carman (45) uses the specific surface S_o of the particles in the cake (cm^2/cm^3) to define this term.

$$R' = \frac{5 S_o^2 (1-\varepsilon)}{g \rho_1 \varepsilon^3} \qquad (52)$$

where

ρ_1 is the density of the cake solids in gm/cm^3.

Hoffing and Lockhart (47) concluded that the resistance to flow is a function of the surface area of the filter media particles instead of the diameter of the filter media particles. Using Equation 52, they calculated the specific resistance of some cakes which agreed very closely with the values obtained by actual constant pressure filtration results. Good correlation was not obtained when the equation containing the particle diameter term was used.

Carman (48) calculated the surface area of steel balls using the filtration resistance with the help of an equation similar to Equation 52. The values so calculated agreed very

closely to the theoretical values.

Tiller (13) developed equations for the determination of specific resistance which are similar to Equation 47 for different filtration processes such as constant rate with and without precoat, and constant pressure with and without precoat under several septum conditions. He also considered the compressibility of the cakes by assuming varying porosity with increasing pressure in constant rate filtration and changing septum resistance in constant pressure filtration. These equations remain only of theoretical interest as no methods were presented to evaluate these variations in a practical situation.

Precoat Filtration (Constant Pressure)

Precoat filtration under constant pressure has seldom been used in Sanitary Engineering practice in the area of potable water filtration. The use of vacuum filtration without precoat in the filtration of sewage sludges, however, is quite common. In vacuum filtration the driving force is obtained by creating a vacuum on the delivery side of the system. The inlet side is open to atmospheric pressure. The net driving head would, therefore, be equal to the negative head created by the vacuum. From a conceptual standpoint, then, the equations developed for constant pressure filtra-

tion will be exactly applicable to the vacuum filtration. The operation of a vacuum filter has been discussed elsewhere (44). The use of a precoat does not make any difference whatsoever in the application of the filtration equations. The precoat only increases the initial septum resistance, R_f.

Filtration resistance with no regard to physical parameters

We can rewrite Equation 43 for non-compressible cakes so that

$$\frac{dV}{dt} = \frac{A^2 P}{\mu(wVR' + AR_f)}$$

or

$$\frac{dt}{dV} = \frac{\mu wVR' + \mu AR_f}{A^2 P}$$

$$= \frac{\mu wR'}{A^2 P} V + \frac{\mu AR_f}{A^2 P} \qquad (53)$$

Therefore, $\frac{dt}{dV}$ versus V will plot as a straight line. From the slope of this line, which will be equal to $\mu wR'/A^2 P$, R' can be calculated. Let the slope be equal to K.

Then,

$$K = \frac{\mu wR'}{A^2 P} \qquad (54)$$

or

$$R' = \frac{K A^2 P}{\mu w} \qquad (55)$$

where

R' = specific resistance of the cake on the basis of the unit weight of diatomite in the cake $[LF^{-1}]$

K = slope of dt/dV versus V plot $[TL^{-6}]$

A = area of the filter cake $[L^2]$

P = pressure loss across the cake or the total driving pressure if the septum and the precoat have a negligible resistance $[FL^{-2}]$

μ = absolute viscosity of the fluid $[FTL^{-2}]$

w = weight of the filter aid in a unit volume of the water filtered $[FL^{-3}]$.

From Equation 19, and since $w = \frac{\gamma_w C_D}{10^6}$

$$\beta[L^{-2}] = R' \frac{\gamma_w}{10^6}$$

$$= \frac{K A^2 P}{\mu w} \frac{\gamma_w}{10^6}$$

$$= \frac{K A^2 P}{\mu C_D} \qquad (56)$$

Alternatively, Equation 53 can be integrated.

$$\frac{dt}{dV} = \frac{\mu w R'}{A^2 P} V + \frac{\mu A R_f}{A^2 P}$$

or

$$dt = \frac{\mu w R'}{A^2 P} V \, dv + \frac{\mu R_f}{A P} dv \, . \qquad (57)$$

Integrating the above equation,

$$\int_o^t dt = \int_o^V \left(\frac{\mu w V R'}{A^2 P} + \frac{\mu R_f}{A P}\right) dV \tag{58}$$

$$t = \frac{\mu w R'}{2A^2 P} V^2 + \frac{\mu R_f}{A P} V \tag{59}$$

$$\frac{t}{V} = \frac{\mu w R'}{2A^2 P} V + \frac{\mu R_f}{A P} \tag{60}$$

Thus, a plot of t/V versus V will also provide a straight line with slope equal to $\mu w R'/2A^2 P$. Let this slope be equal to K'. Comparing K' with K, K' = K/2. (60a)

Now,

$$K' = \frac{\mu w R'}{2A^2 P} \tag{61}$$

or

$$R' = \frac{2K' A^2 P}{\mu w} \tag{62}$$

and

$$\beta = \frac{2K' A^2 P}{\mu C_D} \tag{63}$$

These equations have been used in several studies to calculate filter cake resistance (25, 49, 50, 51). Equation 59 also suggests that the time of filtration, t, plotted against the volume of filtrate, V, will give a parabola.

Thus, the specific resistance of the filter cake can be determined from the results of a constant pressure run. The

only important point to be made here is that the pressure P to be used in Equations 55 and 63 is the pressure loss across the cake and not the total pressure. However, P will be equal to the total pressure if the septum and its precoat have a negligible resistance.

Filtration equation based on physical parameters

All of the equations in the foregoing section defining the specific resistance in terms of the physical characteristics of the cake will be equally valid for constant pressure filtration. In fact, no assumption was ever made in the development of these equations that the filtration will be carried out at a constant pressure or a constant rate. The equations only include the physical characteristics of the cake and the filter aid such as the bulk density of the cake, the porosity, the average particle density and the average surface area of the particle, and were derived for a particular instant during filtration when the rate of flow was q and the head loss H.

It was assumed, however, that the porosity of the cake is constant irrespective of the pressure. This assumption implies an incompressible cake. The implication of this assumption will be discussed later in this thesis.

PURPOSE AND SCOPE OF THIS STUDY

It was stated earlier that economy is the main factor that dictates the design of any engineering operation. In the filtration of water this means the ability to produce the desired quality of water with the minimum cost. It was also mentioned earlier that the filtration cost depends upon several factors such as the costs of labor, power, filter aid and the plant itself.

The most economical design will find the optimum combination of these costs. This is possible only if the filtration is carried out at an optimum combination of unit flow rate, body feed concentration of the most suitable grade and type of filter aid, and terminal head loss, since each of these factors affects one or more of the above costs. Another important objective, of course would be to protect the quality of the filtered water. The use of coarse grades of filter aid might be advantageous because of a higher porosity resulting in a lower head loss. This might save on the cost of labor and backwashing, but the coarser grades of the filter aid are costly to buy. Thus, the filter aid cost component may be higher for the coarser grades of filter aid, but may provide savings in other ways. The situation will be different for the finer grades of the filter aids. In some

cases the quality of the filtered water may be better using the finer grades because of a higher straining and adsorption potential of the fine particles of such filter aids.

Whereas it is possible to optimize all the filtration variables in a new plant, this flexibility may not be possible in an already existing plant where some plant components might have been already purchased, such as the filters and pumps, thus fixing the terminal head loss and the unit flow rate. In such cases an engineer should still optimize the remaining filtration variables. Whatever the case may be, the principles and bases of plant optimization are essentially the same. A computer program, like POPO with appropriate modifications, might be useful. Even manual calculations, although tedious and monotonous in some cases, will be worth the effort since they do help in reducing the overall filtration cost.

The cost of the filter aid associated with least cost filtration is the most important of the various filtration costs stated above and perhaps the most difficult to determine. The unit cost of the plant, power and labor are relatively easy to calculate. The cost of the filter aid is also important for a different reason; it greatly affects the other costs.

An engineer should be able to predict the least cost combinations in advance of the specification of the filtration

equipment. It is therefore desirable to be able to predict the course of any field filtration run from simple tests in the laboratory, since the choice of an optimum concentration of the most suitable grade and type of filter aid is most crucial in least cost filter design. In other words, it is desirable to predict the course of head loss increase while filtering a certain water through any type of precoat filter at specified conditions of flow rate and body feed concentration (C_D) using a particular filter aid. The engineer should also be able to determine the quality of the filtered water which he should expect to get from a full-scale filtration plant. An examination of the filtration equations (Figure 2) and the equations in the Literature Review chapter reveal that the specific filter cake resistance (β-index) is the parameter which needs to be evaluated.

Such a systematic approach to the design of a precoat filtration plant has not been attempted until recently. Due to this some of the water supply regulatory agencies were reluctant to use this method for water filtration. Baumann (49) studied all municipal filtration plants using precoat filters in the United States in 1957. He sent a questionnaire to health departments of all the states to determine their attitude, experiences, and opinions regarding such filtration plants. The responses indicated that six states favored adoption of diatomite filters, 10 states

stated they would approve such installations with reservation, 20 states disapproved, and the rest stated they had no fixed policy on the subject. States approving such plants with reservations generally stated that diatomite filtration plants should be used only on unpolluted well supplies. The objections listed were: lack of understanding of the process, lack of sufficient data in the field, and unsatisfactory performance of some of the existing municipal plants. The unsatisfactory operation of such plants was attributed to poor design and operation due to lack of knowledge of the basic principles involved.

A precoat filter pilot plant was built at Iowa State University and the filtration resistance of several synthetic waters, using several commercially available diatomite and perlite filter aids, has been found (3, 6, 7, 8, 23, 26, 29, 30, 31, 32, 33) by conducting several thousand filtration runs. On the bases of these data, filtration equations were developed by Dillingham et al. (23, 29). This pilot plant is expensive to build, occupies a large laboratory space, and the tests are time consuming. The details of the construction and operation of the pilot plant are presented elsewhere (3). In the operation of the pilot plant, large quantities of the filter aid and test water are needed. These factors prohibit the widespread use of the pilot plant for the determination of the filtration performance

prediction equations using practical waters with fluctuating water quality. The use of the pilot plant, perhaps because of the above limitations, has so far been confined to research purposes only and it has not been used extensively to predict filtration performance and to optimize plant operation in the "real world" involving actual plant operation. The only method available to a precoat filtration design engineer today is to run a full-scale plant fixing the variables on the basis of such experience and changing them as his background, experience, common sense, and familiarity with the situation dictate. In other words, the practical value of the filtration equations developed has yet to be demonstrated in the water works industry.

There is, therefore, a very strong need to devise a simple apparatus or a simple theoretical method that can be employed to determine filter cake resistance, the β-index, for waters actually encountered in the field. This apparatus should be simple enough to be used successfully by an average operator. The apparatus should be inexpensive to build, should be small and handy, and should predict the results using a small quantity of the filter aid and test water. The development of a simple apparatus or a technique for this purpose is vital to the future of precoat filtration itself. It is also essential that the recommended apparatus should give reproducible results under identical filtration

conditions and give accurate prediction of the filtered water quality.

The author made about 300 runs on the pilot plant using an iron bearing (Fe^{+++}) water prepared in the laboratory by adding $FeCl_3 \cdot 6H_2O$ (8 mg/l) to university tap water in a 130-gallon capacity mix tank on a continuous basis (3). It was therefore decided that the best starting point in this study would be to use the same water and one of the same filter aids with the proposed apparatus. Good correlation of the values of the filter cake resistance so obtained with those obtained earlier using the pilot plant under identical conditions would indicate that the new, proposed apparatus was adequate.

If such an apparatus could be successfully designed, the study was to be expanded to determine whether the specific resistance of filter cakes containing the same ratio of suspended solids to body feed was in fact dependent on the concentration of solids in the raw water, as observed and reported by Regunathan (32) to be the case for sodium montmorillonite suspensions. To answer this question more definitely, different types of synthetic suspensions such as a Ball Clay bearing water[1], a sodium montmorillonite

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[1]A product of Old Hickory Clay Co., P. O. Box 271, Paducah, Kentucky.

bearing water[1], and an iron-bearing water of the type described previously were to be tested. The detailed procedure for preparation of these synthetic waters will be given in a subsequent section.

Al-Khafaji (50) built a small constant-pressure apparatus and used it to determine the cake resistance of clay bearing waters without the use of any filter aid. This apparatus appeared to have a potential for use in this study and its use was considered in some detail:

The specific objectives of this study were:

1. The prime objective of the thesis was to develop a bench-scale, laboratory apparatus and test procedure which would require a minimum quantity of the raw water (5-10 gallons) and filter aid to evaluate accurately and reproducibly the quality of the filtered water and the specific cake resistance that would be obtained filtering the water in a full-scale plant in the field. This objective was successfully accomplished by:

   a) A review of the literature was made to determine filtration practices in other industries and to develop suitable filtration equations which would provide knowledge of which filtration parameters had to be controlled and measured.

   b) This review of the literature indicated that

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[1]Wyoming Bentonite or Black Hills clay, a product of International Minerals and Chemical Corporation, Old Orchard Road, Skokie, Illinois.

the specific cake resistance index, β, was theoretically identical for cakes formed under either constant-rate or constant-pressure filtration cycles. Also, the chemical industry had developed constant pressure techniques for evaluating cake resistance of concentrated slurries. Al-Khafaji (50) had adopted these techniques, apparently successfully, to determine cake resistance of clay-bearing waters without the use of filter aids. Accordingly constant-pressure apparatus and techniques were developed to determine whether they could be used to predict accurately the filtered water quality and cake resistance which would be obtained in a practical constant-rate filtration cycle using very dilute suspensions. Subsequently, small-scale, constant-rate apparatus and procedures were developed.

c) The accuracy and reproducibility of cake resistance values using the bench-scale constant-pressure and constant-rate apparatuses and procedures were judged by comparison of similar values obtained using the much larger scale constant-rate pilot plant. These comparisons were made using the filters to remove three different solids from suspension, an iron-bearing water, a Ball Clay bearing water, and a Black Hills-clay bearing water.

2. A secondary objective of the thesis was to make use of the procedures and apparatus developed to explain some of the anomalies observed by various investigators in the operation of the pilot plant. The anomalies studied were:

a) In previous studies and in the filtration equations development, the assumption was always made that no suspended solids were carried through the cake into the precoat and therefore any head loss - time characteristics at the beginning of a run were due only to the initial dilution effect. A study was conducted to evaluate the validity of this assumption.

b) The literature review of constant-pressure filtration results appeared to indicate that cake resistance was frequently concentration dependent. Some of the results collected with the constant-rate pilot plant (29, 32) also appeared to suggest a similar conclusion. A study was therefore conducted to evaluate the validity of the previous observations.

LABORATORY APPARATUS AND PROCEDURES

General

It is evident from the Literature Review that there are three possible methods available which might be used to predict filter cake resistance:

1) On the basis only of measurements of the physical characteristics of the filter aid, the suspended solids, and the cake. These characteristics include the porosity of the cake, the average diameter and surface area of the particles constituting the cake, and bulk density of the cake. (Equations 46 and 52.)
2) On the basis of filtration runs made under constant pressure using the desired conditions of body feed concentration of the specified filter aid using the raw water to be encountered in the field. (Equations 55 and 63.)
3) On the basis of filtration runs made under constant rate conditions simulating field filtration conditions as closely as possible. (Equations 2, 4, 48 and 49.)

It was stated that the equations in the first category, i.e., those which define the specific resistance in terms of the physical characteristics of the filter aid, suspended

solids and the cake, possess a merit over the equations in the other two categories in that these include the parameters which are more basic and fundamental. It is pertinent that a more detailed discussion of these equations be included at this point to fully understand the limitations of these equations, so that the scope and direction of this study can be clearly decided and appreciated.

These equations are:

$$z_c = \frac{K'(1-\varepsilon)^2}{36(v_p/s_p)^2\varepsilon^3}\frac{1}{\gamma_d} \qquad (46)$$

$$z_c = \frac{5\ S_o^2\ (1-\varepsilon)}{g\ \rho_1\ \varepsilon^3} \qquad (52)$$

The literature abounds in a whole series of such equations and perhaps no one equation is better than any other. The limitations of the above two equations, therefore, are valid for all similar equations to all intents and purposes.

In most of these equations, the cake porosity appears as $(1-\varepsilon)^2/\varepsilon^3$ and is referred to as a resistance factor, therefore, $\varepsilon^3/(1-\varepsilon)^2$ is referred to as a permeability factor. The permeability factor is extremely sensitive to the value of ε as shown by the computations in Table 1.

This means that we must be able to calculate or determine the value of ε with great accuracy if this method

is to be depended upon for the prediction of cake resistance. A slight error in the determination of ε (say 0.75 instead of 0.78) will produce highly significant errors ($\frac{10-6.7}{10} \times 100 = 33\%$) in the prediction of the filter cake resistance and thereby in the prediction of head loss in the filtration run.

Table 1. Variation of permeability factor, $\varepsilon^3/(1-\varepsilon)^2$, with porosity ε

	ε	$\varepsilon^3/(1-\varepsilon)^2$
clean, perlite diatomite porosity	0.90	72.9
	0.85	27.3
	0.80	12.8
	0.78	10.0
	0.75	6.7
	0.70	3.8
	0.65	2.2
	0.57	1.0
	0.50	0.5

It is extremely doubtful that the porosity, ε, can ever be determined in a dirty cake consisting of both the suspended solids and the body feed with the desired accuracy. This might be less difficult in a clean cake but the determination of porosity of the cake is still not easy enough to be accomplished by a simple and a quick test. Tiller (13) used a consolidometer to determine the porosity and assumed a suitable value of the specific gravity in his calculation of porosity. A good prediction of ε would

require the determination of the specific gravity of a suspended solids - body feed cake separately. It is questionable whether the value of the porosity determined using a consolidometer would compare favorably with the porosity of the cake laid in actual filtration.

Carman (45) presented an excellent discussion about the variation of porosity in a bed consisting of spherical particles laid in different arrangements. He also presented some experimental observations showing the porosity distribution in actual beds with randomly packed unisize spherical glass beads. He showed that even in such beds, which are more uniform and isotropic than actual filtration beds consisting of particles of different sizes and shapes, the porosity is not constant throughout the bed and the variation is significant. Based on experimental observations, he cited the porosity variation in beds formed by using 50 percent binary mixtures of spheres in varying size ratios. Carman's discussion is interesting and points out clearly the impossibility of specific resistance determination by these methods.

The determinations of other parameters in Equations 46 and 52 are not easy either. Neither the particles of the filter aid nor those of the solids are unisize and, in fact, contain a wide range of particle sizes. In addition, no two methods used to find particle size distribution, such as the hydrometer analysis and the Coulter counter method, are likely

to give the same results (52). It is also doubtful if the bulk density of the dirty cake can be determined with reasonable accuracy. The bulk densities of clean precoat cakes composed of filter aid only were determined in another study (3) at Iowa State University. The method consisted of filtering a filter aid suspension in distilled water (about 10 gms in one liter) through a millipore filter paper under pressure and retaining the filter aid to form the cake. After the filter aid was deposited on the filter paper, the pressure was removed and the thickness of the cake was measured. The cake bulk density was calculated from the dry weight and the volume of the cake.

This method might give incorrect results for at least two reasons: 1) removal of pressure and thus cessation of the flow may disturb the cake and 2) 10 gms of the filter aid in one liter of suspension is an unrealistic, highly concentrated slurry for purposes of water filtration and the bulk density so determined may be different from that in the cake formed with a realistic dilute slurry.

It is easy to visualize that the determination of the surface area, S_o, is also a difficult problem. Attempts have been made by some researchers (47, 48) to determine the specific surface from permeability tests, but it is rather a two step procedure to find S_o from the permeability tests and then to use this value of S_o to calculate the permeability

or resistance. Moreover, such attempts have been confined only to clean cakes and their applicability for dirty cakes are questionable.

In addition, Equations 46 and 52 do not account for certain phenomenon so often reported in the literature, namely electro-osmosis (36, 40, 42, 43), and the dependence of specific resistance on the concentration of solids in the slurry (36, 38, 39). These phenomena have been already discussed in detail. It seems clear that equations for representing cake resistance based on measurement of the physical characteristics of the cake solids alone are extremely inadequate to predict filtration resistance in a practical application. In addition to being inadequate, the evaluation of the parameters involved in such equations is difficult and perhaps impossible to accomplish with reasonable accuracy. The theoretical merit of these concepts is, however, unquestionable.

It was concluded, in the light of these observations, that the only practical method which could be used to determine or to predict the filter cake resistance is to perform actual filtration runs on a small scale pilot plant simulating field conditions as closely as possible.

Al-Khafaji (50) designed a small scale constant pressure filtration apparatus and used it to determine the cake resistance of some synthetic waters without using any filter

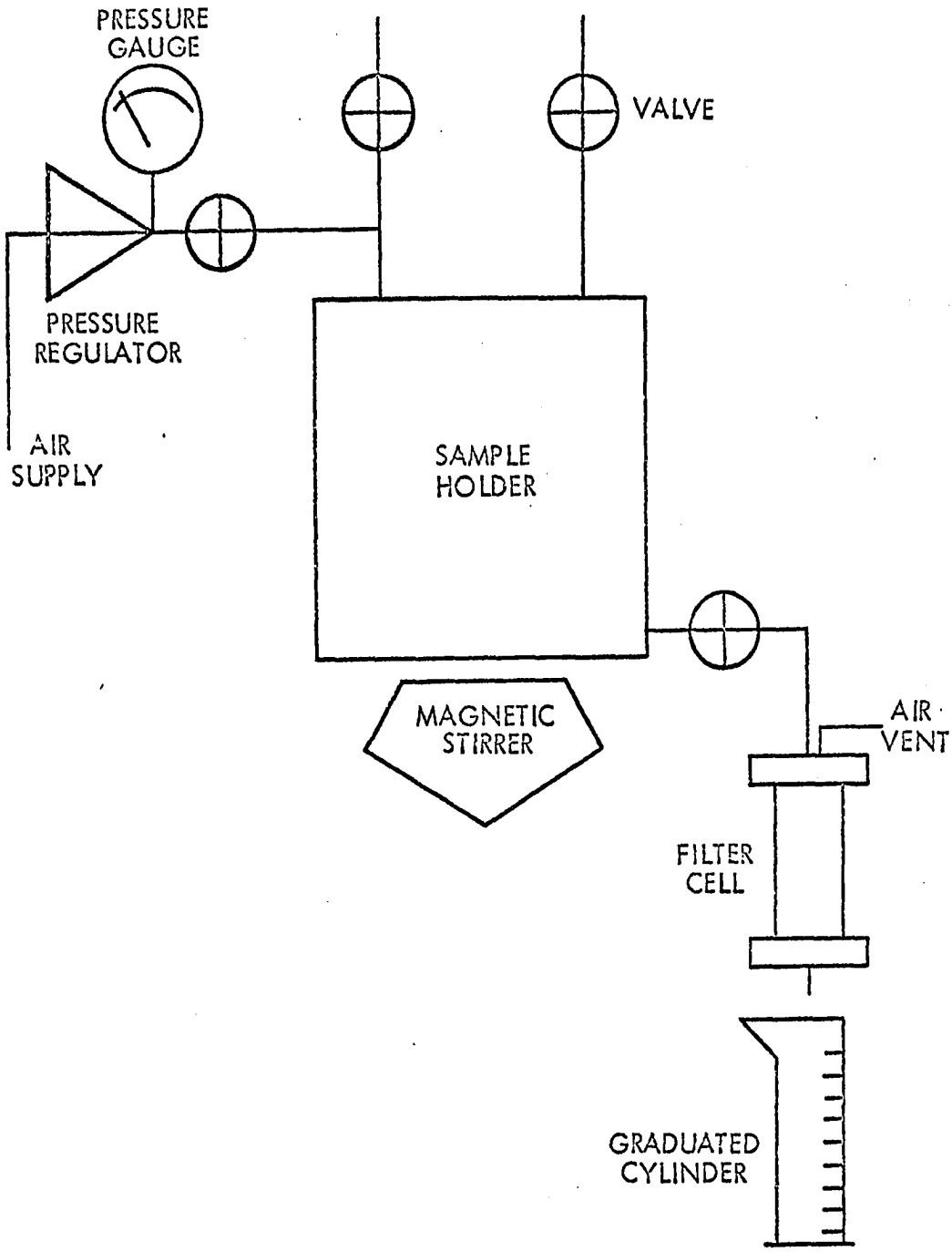
aid. The results of his study could not be tested against similar results obtained on practical installations because such waters are seldom filtered without the addition of body feed. The use of such an apparatus is not new, a similar one having been used by Carman (53) as early as 1934. However, such apparatus has seldom been used to predict filtration resistance in situations encountered in potable water filtration practice, where the raw water contains a relatively low concentration of suspended solids (20-200 mg/l), body feed is added to the raw water, and the filtration is invariably carried out at a constant rate. Due to the simplicity of this apparatus and apparent indication of its successful use, it was decided to explore the use of this apparatus as a first step in this study.

The detailed description of this apparatus and the procedure used in its operation are discussed in the following section.

Laboratory-Scale, Constant-Pressure Filtration Apparatus

A schematic diagram of the typical constant-pressure filtration apparatus used in this study is shown in Figure 3. It consists essentially of a lucite sample holder to house the water sample to be filtered, a lucite filter cell with a filter septum consisting of a millipore filter paper

Figure 3. Schematic diagram of the constant pressure filtration apparatus used in this study


PRESSURE GAUGE
VALVE
PRESSURE REGULATOR
AIR SUPPLY
SAMPLE HOLDER
MAGNETIC STIRRER
AIR VENT
FILTER CELL
GRADUATED CYLINDER

and its support. The system was connected to an air supply line to provide a filtration driving force and the air pressure was controlled by a means of a pressure regulator and measured both by a pressure gauge and a mercury manometer. This apparatus, therefore, was suitable for making constant pressure filtration runs through flat septa. The sample holder was mounted on top of a magnetic stirrer to insure mixing of its contents. The filtrate was collected in a graduated cylinder placed near the outlet end of the filter cell.

Millipore filter papers[1] of 37 mm diameter and 0.45 micron pore size were used to filter the suspension consisting of the raw water and the body feed. After each run the used filter paper was discarded and a new filter paper substituted for the next run.

It was recognized that in order to measure filter cake resistance a cake must be formed in the early part of the run. This could be achieved in two ways:

1) By using a very small diameter (say 3/8-1/2 inch) filter cell. This would help in forming a thick cake with relatively less volume of the suspension in the early part of the run.
2) By using a large sample holder (5-10 liters capacity). It was thought that by wasting 2-3 liters of suspension in forming a cake, the remaining

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[1] A product of Millipore Corporation, Bedford, Massachusetts.

suspension would still be adequate for conducting the run.

Preliminary investigations indicated that neither of the above methods was likely to be successful. The wall friction effects were so significant in the smaller filter cell that it was difficult to form a smooth cake resulting in erratic data. The problems experienced by using the large sample holder were 1) the contents could not be kept well mixed, 2) the runs were very long (for example, 1-2 hours) changing the suspension characteristics throughout the run, and 3) the flow rate diminished to too low a level to be measured accurately.

It was, therefore, decided that a one liter capacity sample holder and about a one inch diameter filter cell[1] were optimum. The complete working drawings of the different parts of this apparatus are presented elsewhere (54). The apparatus used by Al-Khafaji (50) had nearly the same dimensions.

The apparatus was modified slightly towards the end of the study. Two sample holders, each of one liter capacity, were connected to a common filter cell and provision was made to pressurize each or both of them at the same time. See

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[1]The filter cell used had a diameter of 1.065 inches (area = 0.891 inch^2) and depth of about three inches for the cake.

Figure 4. Using this apparatus, the run could be continued as long as needed without some of the problems associated with the use of a single large sample holder. This could be done by filling one sample holder at a time, continuing the run till the contents of this sample holder were close to exhaustion, filling the second sample holder with a fresh suspension and switching over the filtration to the second sample holder with the help of an appropriate stop cock provided in the line.

Procedure

The raw water sample (one liter) was added to the sample holder. The exact amount of body feed to be used in the test was weighed out using an analytical balance[1] and added to the raw water. The suspension was kept under continuous mixing using a magnetic stirrer. The lid of the sample holder was screwed on. The filter cell with a new filter paper was then connected to the system. The suspension, which was under atmospheric pressure in the sample holder, was allowed to flow to the filter cell using a bleed through valve to expel the air in the filter cell.

The pressure regulator was set to the desired pressure,

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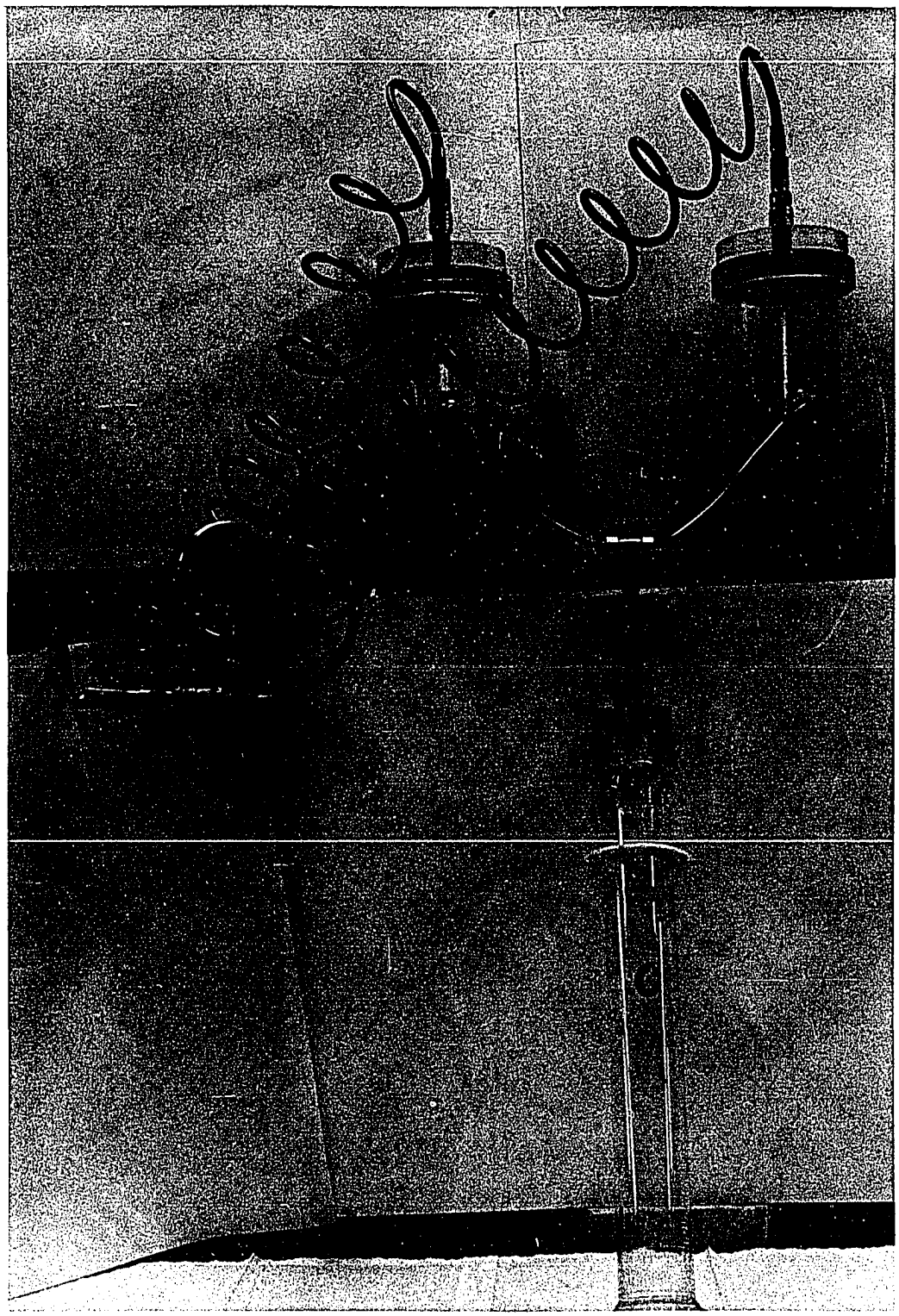
[1]Type H15, capacity 160 gms, manufactured by Mettler Instrument Corporation, Hightstown, New Jersey.

Figure 4. A general view of the modified constant pressure apparatus used in this study



but the exact pressure reading to be used in the cake resistance calculations was read on the mercury manometer. The filtration process was then started by connecting the compressed air line to the top of the sample holder using an easy push through arrangement. Thus, the suspension was forced to flow through the filter cell producing a filter cake at the surface of the filter paper. As the filter cake deposited, the flow rate kept declining. The filtrate was collected in a graduated cylinder placed in a tilted position (see Figure 4) so that the filtrate traveled along the walls of the cylinder without causing a splash, enabling accurate determinations of equal increments of the filtrate volume.

After about 100-200 ml (much more in some cases) of the suspension had filtered through and had formed a cake on the surface of the filter paper, time increments required to produce successive equal increments of filtrate volume were observed. The data were plotted in the appropriate manner and the specific resistance of the filter cake was then calculated by using Equation 55 or 63. (See Appendix A.)

Synthetic Waters

Basically, two types of synthetic waters were filtered in this study:

1) iron bearing waters

2) clay bearing waters, using either
   a) Ball Clay, the essential component of which is kaolin, or
   b) Black Hills Clay, the essential component of which is sodium montmorillonite.

The preparation of these waters is discussed in the following pages.

Iron bearing waters

This water was prepared in exactly the same way as in an earlier study (3). This was done so that the results of filtration (β-indices) obtained on the new small-scale, constant-pressure apparatus could be compared without any bias to the results obtained on the pilot plant.

A stock solution of ferric chloride was prepared by adding 2500 grams of $FeCl_3 \cdot 6H_2O$ and 350 ml of concentrated sulfuric acid to 18 liters of distilled water. The purpose of the acid was to dissolve the ferric chloride completely in the distilled water. Similarly, a copper sulfate stock solution was prepared by dissolving 180 grams of $CuSo_4 \cdot 5H_2O$ in 18 liters of distilled water.

The concentrated iron solution was then fed to a mix tank (36 inches in diameter and 32 inches in depth) through a capillary tube at a drip rate calculated to yield the desired concentration of iron. The stock copper solution

was also fed through a capillary tube at an appropriate drip rate so as to add 1/2 mg/l of Cu^{++} for every 8 mg/l of Fe^{+++}. The function of copper was to precipitate the iron completely (55).

Tap water[1] was added to the mix tank at a constant rate of 7 gallons per minute (gpm) using a Dole flow control valve to regulate the flow. The temperature of the water was maintained at about 22° C. A thermostat[2] was placed in the mix tank to actuate a relay which in turn opened and closed a solenoid valve on the hot water line when the temperature dropped below the desired 22° C level. The mix tank had a 2-inch diameter overflow pipe provided so that the synthetic water could be prepared on a continuous basis.

The solution in the mix tank was mixed mechanically by a rotating paddle with a peripheral velocity of 2.5 feet per second. An air diffuser placed in the bottom of the tank provided for supplemental aeration of the mix tank contents to insure complete mixing and to precipitate the iron completely.

Clay bearing waters

Two clays were used in this study, namely Ball Clay

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[1]A complete mineral analysis of the university tap water is presented elsewhere (3).

[2]Thermoswitch-thermostat, Fenwal Inc., Ashland, Massachusetts.

and Black Hills Clay. The procedure for preparation of the synthetic water using either of the clays was exactly the same.

Ten grams of clay were added to an ordinary bucket containing five liters of distilled water. The contents of the bucket were air agitated for 14-1/2 hours, after which period about two liters of the clay suspension were transferred to an Erlenmyer flask mounted on a magnetic stirrer to keep the contents well mixed. Three samples of this stock suspension, each of 100 ml, were removed and dried in evaporating dishes in an oven overnight. The dried samples were weighed and the average clay concentration in the stock suspension was found. Only the approximate concentration of clay (C_s) was known on the day when the filtration runs were made using a particular batch of a synthetic water. The exact concentrations used were known on the following day. Successive dilutions of the stock suspension were prepared for filtration runs using one of the following fluids:

1) tap water filtered through 0.45 micron millipore filter papers
2) tap water -- unfiltered
3) distilled water.

In some filtration runs the stock clay suspension was prepared in a slightly different way. After air agitating the bucket

contents for 14-1/2 hours, the suspension was allowed to settle for 6 hours. Two liters of the supernatant were removed, and used as the stock suspension for successive dilutions.

Thus, a synthetic water designated as "settled Ball Clay in filtered tap water" would mean a water in which the Ball Clay suspension in the bucket was settled for six hours, the supernatant was removed as stock, and successive dilutions of this stock were prepared using the filtered tap water.

Similarly, another synthetic water designated as "unsettled Black Hills Clay in distilled water" would correspond to a water in which the Black Hills Clay suspension was removed to the flask as stock immediately after 14-1/2 hours air mixing and further dilutions of this stock were made using distilled water.

In the work reported, 10 mg of Calgon was added to the bucket to insure a good dispersion of the clay during air agitation.

Analytical Techniques

Iron bearing water

The colorimetric-phenanthroline method was used to analyze for iron. This is one of the standard methods adopted

in the water supply field, is fairly sensitive, and the analysis can be performed rapidly (56). A phenanthroline reagent in a patented powder form[1] dissolves the iron in water instantaneously at ordinary room temperature when 25 mg of reagent are added to 25 ml of the test water containing 0-3 mg/l of iron. If the test water contains iron in amounts greater than 3 mg/l, it may be suitably diluted or an additional scoop of the powder may be added.

The water develops an orange color on addition of this reagent, which obeys Beer's Law. Full development of the color takes about two minutes. A spectrophotometer[2] was used to determine the percent transmittance of light passing through the solution. The wave length best suited for this purpose was found to be 510 millimicrons on the basis of the method explained by Sawyer (57). From the percent transmittance of light determined, the iron concentration in the water sample in mg/l could be determined using a laboratory prepared calibration curve.

Clay bearing water

As explained earlier the concentration of clay in the synthetic water was determined gravimetrically. However, the

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[1]Ferrover, Hach Chemical Company, Ames, Iowa.

[2]Model B spectrophotometer, Beckman Instrument, Inc., Fullerton, California.

effluent quality was monitored in some cases using a turbidimeter.[1]

[1]Hach Laboratory Turbidimeter Model 1860, Hach Chemical Co., Ames, Iowa.

PRESENTATION OF RESULTS

General

As explained earlier, the constant pressure apparatus was used first for prediction of cake resistance which could be compared with the cake resistance obtained using the pilot plant in filtration of iron bearing waters.

This apparatus was used first for several reasons:

1) The constant pressure apparatus had been used by Al-Khafaji (50) and had provided good cake resistance reproducibility.
2) The apparatus is inexpensive (< $100) and its operation is extremely simple.
3) The tests are quick to perform, requiring about 10-15 minutes.
4) Very small amounts of the filter aid and test water are necessary (less than one gm of the filter aid and one liter of water for each run).
5) Attempts, whether successful or not, to predict the performance of constant rate filtration on the basis of the results of constant pressure filtration may help in improving the understanding of the cake filtration process. This may tie up the theories of the two filtration processes, which appear so identical in concept.

Selection of Pressure

An important question to be decided was what pressure should be used in the constant pressure runs. Preliminary runs indicated that iron bleed through in the effluent was significant when pressures higher than 5 psi were used. This observation indicated that significant amounts of iron were being carried out of the filter cake and through the millipore filter septum at the flow rates involved (5 to 10 gpm/sq ft). For example, a run made on iron bearing water (8.14 mg/l as Fe^{+++}) using C535-X[1] as a filter aid (C_D = 177 mg/l) showed an effluent iron concentration of about 2.19 mg/l when a pressure of 151.4 cms Hg was used. An identical run on the pilot plant would have produced an iron free water (< 0.06 mg/l) as was observed in an earlier study (3). Unless the quality of the filtered water in the two apparatuses is comparable, the basis of comparison of the two results is not valid.

Still higher pressures produced a worse effluent. Even at a pressure of 10 psi, some runs using a low filter aid concentration did not produce a good water. However, when a pressure of 5 psi was used, the quality of the effluent was comparable to that obtained on the pilot plant with little

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[1]Sample taken from bag No. X of grade 535 of Celite diatomite, a product of Johns-Manville Products Corporation, Manville, New Jersey.

or no bleed through of iron (< 0.06 mg/l). It was reported in earlier studies (3, 50) that pressures greater than 5 psi produced smooth filter cakes and in the runs with pressures less than 5 psi, the cakes were not uniform and smooth.

On the basis of these observations, a pressure of about 5 psi was considered optimum and was, therefore, selected for making most of the filtration runs. In the runs made on the Black Hills Clay bearing waters, a pressure of about 30 psi was used since these waters were extremely resistant to filtration and would not filter at measurable rates without the application of such a high pressure. The filtered water in such cases was free from turbidity in spite of the high pressure used. Since the addition of the body feed in suitable proportions is likely to produce rather incompressible cakes, it was assumed that the specific resistance would not depend on the pressure.

Precoat Versus No Precoat

In all the filtration runs made on the pilot plant, the septa were precoated using about 0.15 lb per sq ft of the filter aid. On a cursory thinking it would appear that the septum of the constant pressure apparatus (millipore filter paper) should also be precoated likewise for a rational comparison. Preliminary work indicated that the use of a pre-

coat produced a filtered water quality comparable to that produced on the pilot plant even at very high pressures. This precoat was laid by filtering a suspension of distilled water and precoat filter aid through the millipore filter paper.

These considerations suggested that the use of a precoat in the constant pressure apparatus should be investigated thoroughly. The results of a few runs made on the iron bearing water using a precoat were highly erroneous. The increase in time intervals for the filtration of successive equal volumes of the water were not at all uniform resulting in a complete distortion of the plot of $\Delta T/\Delta V$ versus V or T/V versus V from linearity. On examination of the cake and the operation, the departures observed were attributed to the following reasons.

1) After precoating, the impact of the raw water disturbed the precoat significantly when the filtration run was started. This disturbance was noticed as an unevenness in an otherwise even precoat immediately on the start of the run.
2) The removal of pressure used in precoating after the completion of this operation caused a great deal of cake disturbance and vibration of the precoat. This removal was necessary to fill the sample holder with the test water to be used for filtration. The vibra-

tion of the precoat could be clearly observed on the withdrawal of the pressure. Even the cessation of flow through the cake (precoat in this case) has been reported to cause vibrations and disturbance to the cake (39). Preliminary work conducted confirmed this observation.

3) An examination of the cake during and at the end of filtration revealed a penetration of iron floc into the top (about 1/16 inch) layers of the precoat. This could be seen clearly because of the difference in colors of the precoat and the iron floc (white and red, respectively). This might be due to a relatively high impact of the raw water on the precoat, which might have decompacted the top portions of the cake on removal of pressure.

It is possible that the characteristics of the precoat, and therefore the cake laid on it during filtration, may have been distorted or changed as a consequence of one or more of the above factors. The extent of distortion might be different in different layers of the precoat.

Although the modification of the apparatus (Figure 4) alleviated some of these problems by not interrupting the flow and by not removing the pressure after precoating, some problems still continued. A certain amount of the filter aid settled in the pipes and valves during precoating between

the sample holder and the filter cell, and mixed with the raw water during filtration changing the body feed concentration in a way that cannot be determined. Since the weight of the precoat is relatively much higher than the body feed (about 0.45 gm against 0.1 gm or less) in an average run, even a slight amount of settling can produce serious errors. This problem can be minimized by washing the sample holder and the pipe lines using distilled water several times and filtering this water through the precoat until any filter aid, if settled, is washed out of the appurtenances. This will have to be done at the expense of time.

Since the use of the precoat only affects the initial head loss in a constant rate filtration operation and the initial filtration rate in constant pressure filtration (47) without changing the average specific resistance of the cake, it was concluded that the use of the precoat would not serve any useful purpose. This also appears evident from a consideration and understanding of the filtration theory explained in an earlier chapter. On the contrary, it may create problems pointed out in the foregoing pages. It was, therefore, decided that precoat is not at all essential in the accurate determination of the specific resistance of the cake using constant-pressure techniques.

Pilot Plant Runs

Several runs were made on the pilot plant in an earlier study (3) on iron bearing waters (8 mg/l Fe^{+++}) using several grades of diatomite and perlite as filter aid. One of the grades of diatomite used was C535 , which is a medium grade of Celite, a Johns-Manville product. This filter aid was arbitrarily chosen to be used as body feed for the runs. Unfortunately, the entire sample of the filter aid of this grade of diatomite was used up. The only alternative available was to open a new bag of the same grade of filter aid. Since the proof of success of the apparatus lay in a good correlation of the results obtained on this apparatus with those on the pilot plant, it was decided to use a part of the new sample to make about 4-5 new runs on the pilot plant instead of relying on the earlier results, which might be different due to some differences in the manufacturing and processing operations of the filter aids.

Five filtration runs were made on exactly identical water using the new bag of this filter aid (C535-X). The only variable from one run to the next was the body feed concentration, C_D. Using the data of each run, the β-index was calculated on the bases of the appropriate equations for cylindrical septa listed in Figure 2. To do this a computer program, "BID", written at Iowa State University was employed.

A sample of manual calculations showing the determination of the β-index was presented in another study (3).

On the basis of the values of C_s/C_D in each run and corresponding value of β-index, a Beta prediction equation, such as Equation 8, was developed with the help of another computer program,"MAIDS". This equation could also be developed using manual calculations. This equation was comparable to the representative Beta prediction equation developed on the basis of three samples of this filter aid tested earlier (3).

The results of these five runs made on the pilot plant are summarized in Table 2.

About 1 lb of a sample of another filter aid, Gl-1,[1] was left over from the pilot plant runs. It was, therefore, decided that this remaining sample could also be used for filtration runs on the constant pressure apparatus and results compared with those on the pilot plant.

Runs on the Constant Pressure Apparatus

Reproducibility

Although the reproducibility of results on the pilot plant had been reported to be excellent (± 5 percent) in

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[1]Bag No. 1 of Speedex diatomite (3), a product of Great Lakes Carbon Corporation, Chicago, Illinois.

Table 2. Summary of results of the pilot plant runs on iron bearing water using C535-X as filter aid

Run no.	C_s (mg/l)	C_D (mg/l)	C_s/C_D	$\frac{\beta}{10^6}$ ft^{-2}	Remarks
1	8.10	253	0.0320	20.29	Condition of runs
2	8.09	363	0.0223	10.22	1) Precoat = 0.15 lb/ft^2
3	7.91	443	0.0178	6.78	2) Effluent iron concentration = <0.06 mg/l
4	8.06	206	0.0392	30.74	3) Flow rate = 1 gpm/ft^2
5	8.18	153	0.0535	57.02	4) Temperature = 21°C-22°C
$\beta = 10^a (C_s/C_D)^b$					5) γ_d for C535-X = 19.95 lbs/ft^3
a = 10.2232					6) Dilution factor δ = 18/hour
b = 1.9440					
R = 99.978 percent					

earlier studies (3, 32), it was necessary to establish whether the constant pressure apparatus could also give reproducible results under identical conditions.

Several runs were made in duplicate to study this, and reproducibility of results was very good in almost all cases. The results of four such runs, two identical runs on iron bearing waters (Runs 69, 70) and two identical runs on Ball Clay bearing waters (Runs 203, 204), are presented in Tables 3 and 4.

The results in Tables 3 and 4 indicate that the reproducibility of the results on identical runs using the

Table 3. Results of two identical runs on iron bearing waters

Run 69				Run70			
V ml	ΔV ml	ΔT sec	ΔT/ΔV sec/ml	V ml	ΔV ml	ΔT sec	ΔT/ΔV sec/ml
100				100			
200	100	27.1	0.271	200	100	28.5	0.285
300	100	30.5	0.305	300	100	31.7	0.317
500	200	67.0	0.335	500	200	70.3	0.352
600	100	37.5	0.375	600	100	38.1	0.381
700	100	39.3	0.393	700	100	41.5	0.415
800	100	42.7	0.427	800	100	43.7	0.437
900	100	44.0	0.440	900	100	45.6	0.456
1000	100	45.7	0.457	1000	100	47.6	0.476

Condition of runs

C_S = 8.09 mg/l	C_S = 8.09 mg/l
C_D = 803 mg/l	C_D = 799 mg/l
C_S/C_D = 0.01	C_S/C_D = 0.01
Pressure P = 59.5 cm Hg	Pressure P = 59.5 cm Hg
Temperature = 23° C	Temperature = 23° C
Filter aid used = C535-X	Filter aid used = C535-X
Effluent iron conc = 0.06 mg/l	Effluent iron conc = 0.08 mg/l
$\beta = 2.164 \times 10^6/ft^2$	$\beta = 2.164 \times 10^6/ft^2$

Table 4. Results of two identical runs on settled Ball Clay in filtered tap water

Run 203				Run 204			
V ml	ΔV ml	ΔT sec	ΔT/ΔV sec/ml	V ml	ΔV ml	ΔT sec	ΔT/ΔV sec/ml
100				100			
150	50	23.5	0.470	150	50	22.8	0.456
200	50	25.2	0.504	200	50	24.6	0.492
250	50	29.0	0.580	250	50	28.6	0.572
300	50	30.2	0.604	300	50	29.1	0.582
350	50	32.3	0.646	350	50	31.8	0.636
400	50	34.1	0.682	400	50	33.7	0.674
450	50	35.8	0.716	450	50	35.2	0.704
500	50	37.2	0.744	500	50	37.1	0.742
550	50	39.0	0.780	550	50	37.1	0.742
650	100	78.8	0.788	600	50	38.8	0.776
700	50	41.5	0.830	700	100	80.6	0.806

Condition of runs

C_S = 9.55 mg/l

C_D = 60 mg/l of Hyflo Super-cel[a]

C_S/C_D = 0.159

Pressure P = 27.1 cm Hg

Temperature = 26° C

Effluent turbidity = 0

$\beta = 0.162 \times 10^8$ ft^{-2}

C_S = 9.55 mg/l

C_D = 60 mg/l of Hyflo Super-cel[a]

C_S/C_D = 0.159

Pressure P = 27.1 cm Hg

Temperature = 26° C

Effluent turbidity = 0

$\beta = 0.164 \times 10^8$ ft^{-2}

[a]A fine grade of Celite, a diatomite product of Johns-Manville Products Corporation, Manville, New Jersey.

constant pressure apparatus is very good. See Figures 5 and 6 for the plots of $\Delta T/\Delta V$ versus V for these runs. The slope of plots of $\Delta T/\Delta V$ versus V in both the identical runs is the same in Figure 5. Similarly, it was possible to draw only one line for the results of the two identical runs of Figure 6.

The reproducibility in all the duplicate runs was not always perfect, as shown by the results of Tables 3 and 4 plotted in Figures 5 and 6 respectively. In general, however, it was noted that the results of identical runs were close ($\pm$ 5 percent).

Other comments - cake formation

There is a striking difference between the plots of Figure 5 and those of Figure 6. Whereas the plots of Figure 5 are straight lines from beginning to end, the plots of Figure 6 can be divided into two distinct regions. The first region corresponds to plugging and coverup of pores in the millipore septum. At the common point between the two regions, the cake starts forming. The slope of the second straight line, therefore, represents the cake resistance and should be, and was, used for the calculation of specific resistance of the cake. The initial portion of the plot in such a case, when complete septum coverage with cake is taking place, may not be a straight line. Only for

Figure 5. Plots of V versus $\Delta T/\Delta V$ from the results of two identical runs made to check reproducibility of test results (Table 3)

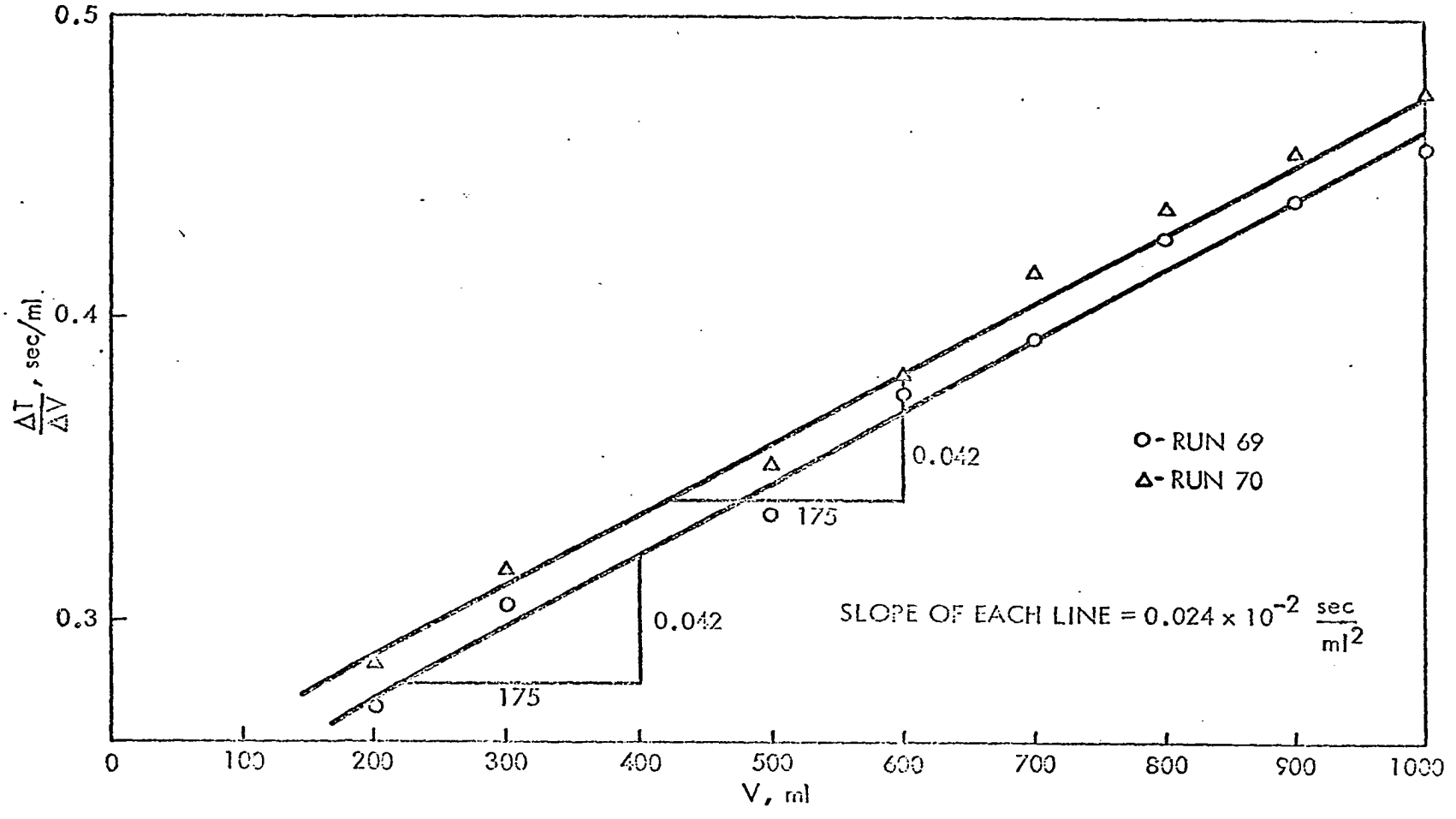
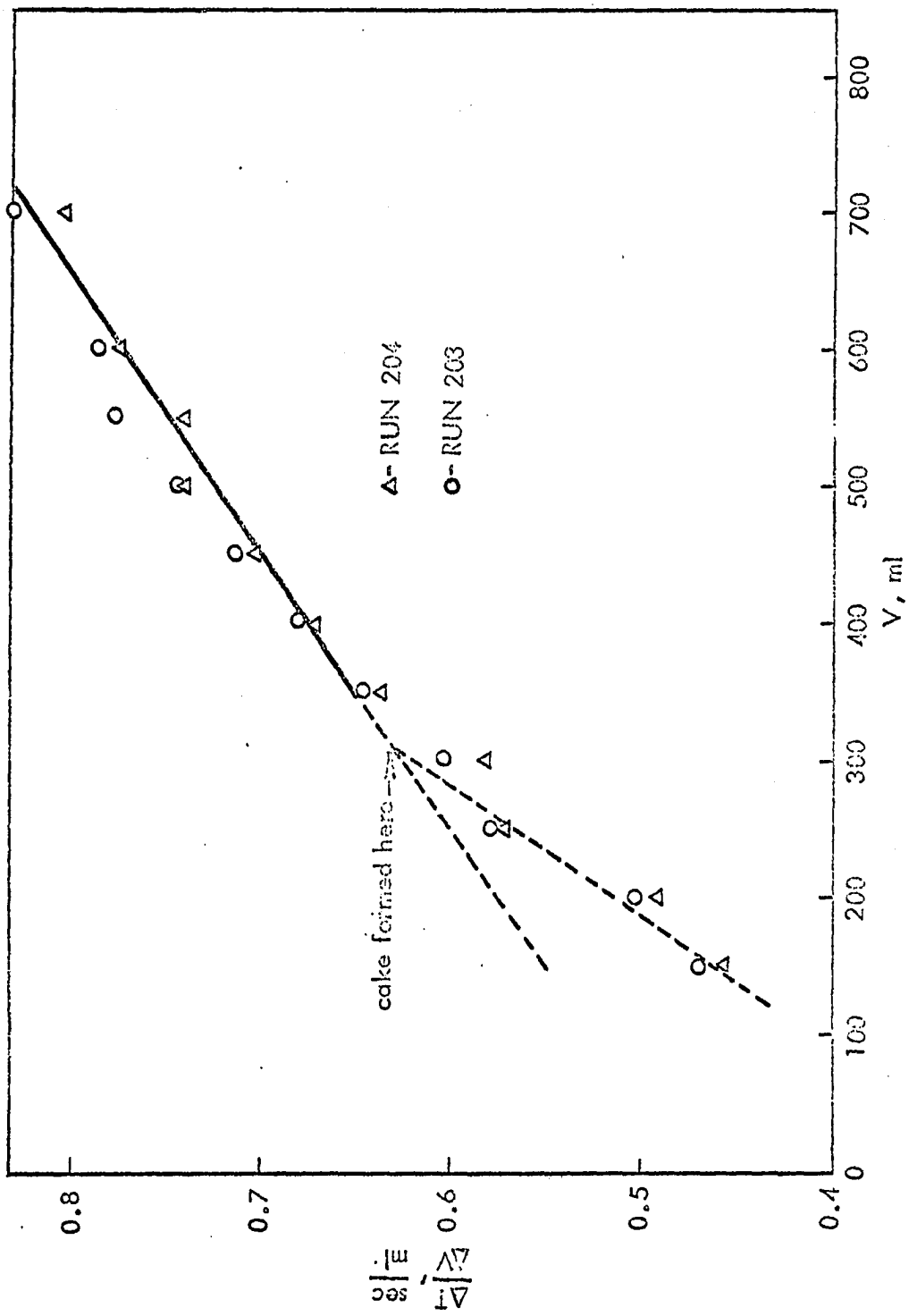

0.5
0.4
0.3
$\frac{\Delta T}{\Delta V}$, sec/ml
0
100
200
300
400
500
600
700
800
900
1000
V, ml
0.042
175
0.042
175
O- RUN 69
Δ- RUN 70
SLOPE OF EACH LINE = 0.024×10^{-2} $\frac{sec}{ml^2}$

Figure 6. Plots of V versus $\Delta T/\Delta V$ from the results of two identical runs made to check reproducibility of test results (Table 4)


Δ- RUN 204
o- RUN 203
cake formed here
V, ml
0
100
200
300
400
500
600
700
800
0.4
0.5
0.6
0.7
0.8
Δt/ΔV, sec/ml

simplicity has it been shown as such.

The reason why the region of septum plugging is not perceptible in the plots of Figure 5 is not difficult to determine. Runs of Figure 5 were made with a suspended solids concentration of 8.09 mg/l and body feed concentration of about 800 mg/l. On the contrary, the runs of Figure 6 were made with a suspended solids concentration of 9.55 mg/l but a body feed concentration of only 60 mg/l. It is therefore conceivable that the particular type of septum, the millipore filter membrane, will be covered uniformly much faster in the first series of runs. It is expected that the septum coverage with cake might be completed by the solids and the body feed filter aid retained from the first 100 ml of the suspension in this case. In the runs using a water with a low solids and body feed concentration, about 350 ml of the suspension were required to cover all the pores in the septum. This behavior of septum coverage was also noticed in the filtration of sugar solutions (58).

This phenomenon of septum coverage will be discussed later in greater length with its implications on the determination of filter cake resistance, β-index.

Comparison with the pilot plant results - iron bearing water

After having insured reproducibility, it was necessary to find out whether the results of the constant pressure

apparatus agree reasonably with the results of the pilot plant. Runs were made on the iron bearing water using C535-X as a filter aid. The data of each run were used to calculate the specific filter cake resistance, β-index. The corresponding values of β-indices for the constant rate filtration on the pilot plant were determined on the basis of the Beta prediction equation developed from the results of the pilot plant (Table 2) by substituting the appropriate value of the ratio of suspended solids concentration to the body feed concentration (C_s/C_D). In five filtration runs, G1-1 was used as a filter aid. The values of β-indices for constant rate filtration under identical conditions were calculated for such cases using the Beta prediction equation developed for exactly the same filter aid in another study (3). This equation is

$$\beta = 10^{a} \, (C_s/C_D)^{b}$$

where

$a = 10.195$

$b = 1.772$

$R = 99.611$

A summary of the results of the constant pressure runs and the corresponding cake resistance values from constant rate filtration runs (calculated from the corresponding Beta

prediction equation) are presented in Tables 5-6 and 7.

Although the results of some of the runs made on the constant pressure filtration apparatus are reasonably close to the corresponding results on the constant rate pilot plant (Runs 355, 356, 357, 362, 363) the difference is too much for most runs. Ordinarily it would be expected that the comparison between the results of the constant rate filtration runs and the constant pressure filtration runs would be better at a low value of C_s/C_D (i.e., using high body feed concentration) because of the early septum plugging in such runs. The results presented in Tables 6 and 7 show an opposite tendency, however, the comparison getting better with increasing values of C_s/C_D. A possible reason for this behavior will be discussed later.

A few runs were made using a pressure of 27.3 cms Hg instead of the higher pressure (50-60 cms Hg) used in most of the runs listed in Table 5-6. The values of β-indices for all these runs, made at both pressures, are plotted against C_s/C_D on a log plot in Figure 7. One single line passes through all the points indicating that the specific resistance was independent of the pressure used in these runs. This implies that the cakes formed are incompressible over the pressure range and body feed range used. Therefore, it can be concluded that the unsatisfactory comparison of the results obtained in constant pressure filtration with the

Table 5-6. Summary of constant pressure filtration runs on iron bearing water using C535-X as filter aid

Run no.	C_s mg/l	C_D mg/l	C_s/C_D	Pressure, P cms of Hg	$\frac{\beta}{10^6}$ ft^{-2} [a]	$\frac{\beta_p}{10^6}$ ft^{-2} [b]	$\frac{(\beta_p-\beta)}{\beta_p} \cdot 100$
353	8.9	800	0.0111	57.7	1.087	2.649	59
358	8.3	800	0.0104	57.7	1.078	2.307	53
354	8.3	400	0.0208	57.7	4.137	8.974	54
355	8.31	160	0.052	57.7	44.69	52.87	15.5
356	8.30	120	0.0692	57.7	87.35	90.78	3.8
357	8.30	80	0.104	57.7	198.8	205.2	3.1
69	8.09	803	0.01	59.5	0.797	2.164	63
70	8.09	803	0.01	59.5	0.808	2.164	63
75	7.95	400	0.0199	59.5	3.044	8.189	63
76	7.72	400	0.0193	59.5	3.060	7.82	61

[a] The values of β-indices were calculated using a computer program. The program was designed to draw a best fit line between the values of T/V and V and then calculate β on the basis of Equation 63. However, manual calculations of β are also quick and simple, as shown in Appendix A. These values are for constant pressure filtration.

[b] β_p = β expected from the pilot plant.

Table 5-6 (Continued)

Run no.	C_s mg/l	C_D mg/l	C_s/C_D	Pressure, P cms of Hg	$\frac{\beta}{10^6}$ ft^{-2} [a]	$\frac{\beta_p}{10^6}$ ft^{-2} [b]	$\frac{(\beta_p-\beta)}{\beta_p} \cdot 100$
359	7.67	800	0.0096	27.3	0.939	2.009	54
360	7.61	400	0.019	27.3	3.216	7.603	58
361	8.12	160	0.0507	27.3	34.2	50.47	32
362	8.03	120	0.067	27.3	73.58	87.70	16
363	7.98	80	0.1	27.3	175.8	190.1	7-1/2

Note: In all these runs the filtered water contained less than 0.1 mg/l iron and thus was considered comparable to the water produced by the pilot plant.

Table 7. Summary of constant pressure filtration runs on iron bearing water using G1-1 as filter aid

Run no.	C_s mg/l	C_D mg/l	C_s/C_D	Pressure, P cms of Hg	$\frac{\beta}{10^6}$ ft^{-2} [a]	$\frac{\beta_p}{10^6}$ ft^{-2} [b]	$\frac{(\beta_p-\beta)}{\beta_p} \cdot 100$
364	8.20	800	0.0102	56.5	1.569	4.721	67
365	8.20	400	0.0205	56.5	5.88	16.0	63
366	8.20	160	0.0512	56.5	53.33	80.91	34
367	7.41	120	0.0617	56.5	78.9	113.2	30
368	7.73	80	0.0967	56.5	176.9	247.7	28-1/2

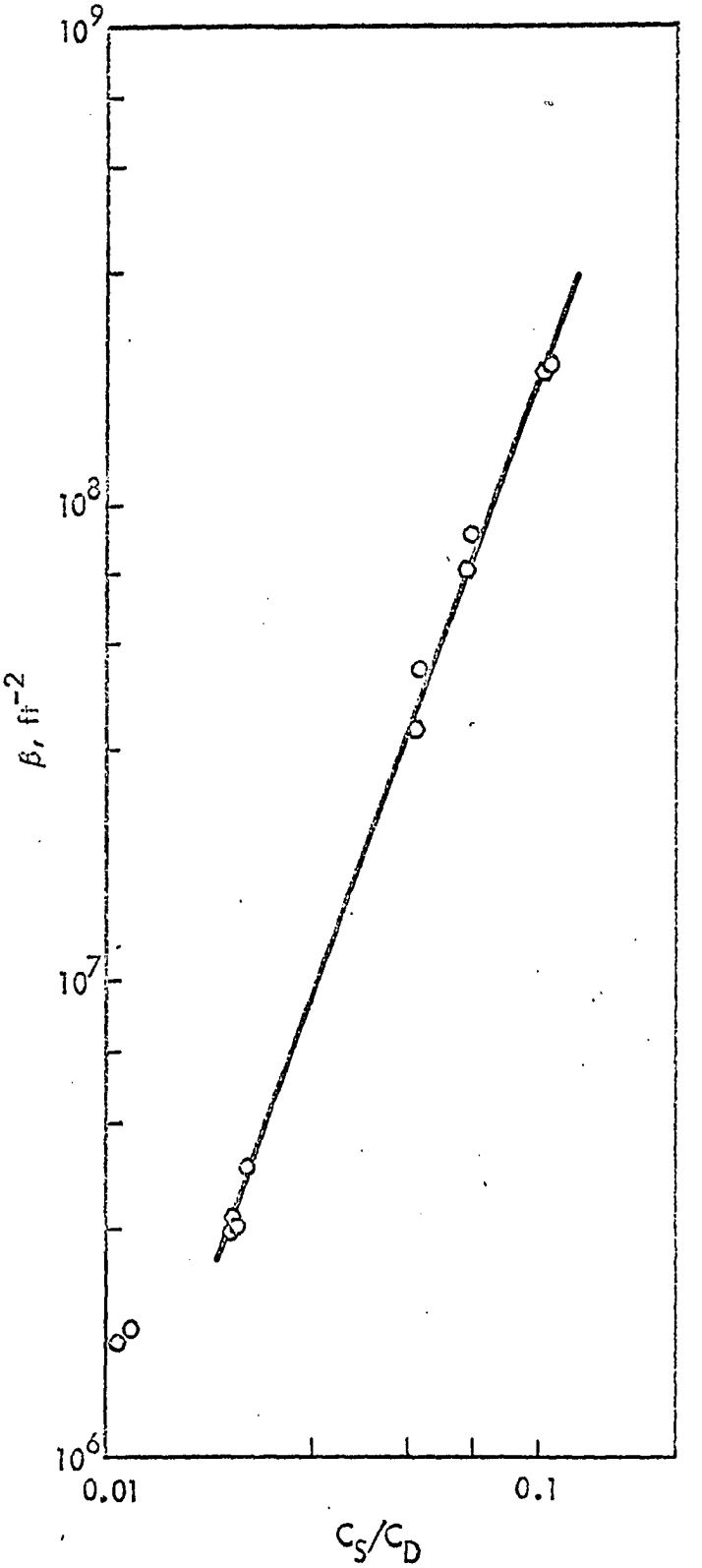
[a] The values of β-indices were calculated using a computer program. The program was designed to draw a best fit line between the values of T/V and V and then calculate β on the basis of Equation 63. However, manual calculations are also very quick and simple as shown in Appendix A. These values are for constant pressure filtration.

[b] β_p = β expected from the pilot plant.

Figure 7. Values of β-indices obtained from the results of constant pressure filtration plotted against ratios of the suspended solids concentration to body feed concentration (C_s/C_D) for iron bearing waters (see results of Table 6)

O indicates runs made at a pressure of 57-59 cms Hg

O indicates runs made at a pressure of 27.3 cms Hg


10^9
10^8
10^7
10^6
β, ft^{-2}
0.01
0.1
C_S/C_D

expected results on the pilot plant cannot be attributed to the use of one particular pressure in preference to any other reasonable pressure that might have been selected.

Appendix A contains an example illustrating the computation of a β-index from the results of a typical constant pressure run.

Specific resistance versus the solids concentration in constant pressure runs

Regunathan (32) observed that the specific resistance decreased with increasing concentration of solids during the filtration of waters bearing Black Hills Clay as an impurity and using Hyflo Super-cel as a filter aid, even though the ratio of solids concentration to body feed concentration, C_s/C_D, was kept the same. Such a behavior was not observed in the filtration of Ball Clay bearing waters. The runs were made by Regunathan on the pilot plant using a constant rate of filtration (1 gpm/ft^2). This observation was contrary to the commonly accepted hypothesis that cake resistance is independent of solids concentration in the realm of precoat filtration as applied to potable water and was attributed to the enormous swelling exhibited by Black Hills Clay when placed in contact with water. It was decided to verify this behavior using the constant pressure apparatus.

Several series of runs were made. In any particular series of runs, a fixed ratio of suspended solid concentration

to body feed concentration (C_s/C_D) was maintained. Thus, runs in any one series were made at different values of C_s and C_D, but keeping the same ratio. In all these runs, Hyflo Supercel was used as the filter aid.

It was found that it is extremely difficult to reproduce a settled clay suspension from day to day in spite of identical procedures used in their preparation. This might be attributed to different settling rates of the clay particles on different days because of varying temperatures in the laboratory from one day to another. It was, therefore, considered necessary that runs in any one series, using settled clay bearing waters, all be made on the same day. This imposed a limitation on the number of runs in any one series. Even the range of C_s to be covered in these runs was limited, in some cases, depending on the amount of stock clay solution available.

The following series of runs were made to study the effect of concentration of solids on the specific resistance, β-index; using constant-pressure techniques.

1) settled Ball Clay in filtered tap water
   $C_s/C_D = 0.39$ (Runs 244-251)
2) settled Ball Clay in distilled water
   $C_s/C_D = 0.37$ (Runs 252-259)
3) settled Black Hills Clay in filtered tap water
   $C_s/C_D = 0.457$ (Runs 228-235).

A summary of the results of the above runs is presented in Tables 8, 9, and 10. The results of these runs are shown graphically in Figures 8, 9, and 10. The results indicate that the specific resistance is the highest at low concentration of body feed (or even the low concentration of suspended solids) for a fixed value of C_s/C_D. As the body feed concentration or the suspended solids concentration increases, the specific resistance rapidly falls, till it assumes a nearly constant value at a high concentration of body feed or suspended solids. Another interesting observation in these results is that the Ball Clay in distilled water had about 3-4 times higher filtration resistance than identical clay suspension prepared in tap water. See results of Tables 8 and 9.

Discussion of Results

The results in the previous section provided strong evidence of the dependence of specific resistance on the concentration of solids in the slurry. Several articles in the literature (36, 37, 38, 39) have indicated this concentration dependence, but the effect was never reported to be as great as observed in this study. In fact, Tiller and Shirato (37) pointed out that the effect of concentration would be significant only with high concentrations of solids in the slurries (30-40 percent solids). The reasons advanced

Table 8. Summary of constant pressure filtration runs on settled Ball Clay in filtered tap water using Hyflo Super-cel as a filter aid (C_s/C_D = 0.39)

Run no.	C_s mg/l	C_D mg/l	Pressure, P cms Hg	$\beta/10^6$ ft^{-2}	Remarks
244	1.94	5.0	27.6	69.7	
245	3.87	10.0	27.6	79.2	
246	7.74	20.0	27.6	61.5	
247	15.48	40.0	27.6	28.3	The effluent was turbidity free in all these runs
248	30.96	80.0	27.6	11.5	
249	61.92	160.0	27.6	6.0	
250	123.84	320.0	27.6	4.9	
251	247.68	640.0	27.6	4.9	

Table 9. Summary of constant pressure filtration runs on settled Ball Clay in distilled water using Hyflo Super-cel as a filter aid (C_s/C_D = 0.37)

Run no.	C_s mg/l	C_D mg/l	Pressure, P cms Hg	$\beta/10^6$ ft^{-2}	Remarks
252	1.84	5.0	27.5	169.6	
253	3.68	10.0	27.5	79.0	
254	7.35	20.0	27.5	74.4	
255	14.7	40.0	27.5	69.6	The effluent was turbidity free in all these runs
256	29.4	80.0	27.5	51.1	
257	58.8	160.0	27.5	29.0	
258	117.6	320.0	27.5	19.7	
259	235.2	640.0	27.5	19.0	

Table 10. Summary of constant pressure filtration runs on settled Black Hills Clay in filtered tap water using Hyflo Super-cel as a filter aid (C_S/C_D = 0.457)

Run no.	C_S mg/l	C_D mg/l	Pressure, P cms Hg	$\beta/10^6$ ft^{-2}	Remarks
228	18.88	100.0	155.0	708	Reproducibility runs
229	18.88	100.0	155.0	776	Mean $\beta/10^6$ = 742
230	37.76	200.0	155.0	287	Reproducibility runs
231	37.76	200.0	155.0	284	Mean $\beta/10^6$ = 285.5
232	75.52	400.0	155.0	202.2	Reproducibility runs
233	75.52	400.0	155.0	217.0	Mean $\beta/10^6$ = 209.5
234	151.04	800.0	155.0	152.0	Reproducibility runs
235	151.04	800.0	155.0	159.0	Mean $\beta/10^6$ = 155.5
					The effluent was turbidity free in all these runs

Figure 8. Variation of β-index with C_D at C_S/C_D = 0.39 in the filtration of Ball Clay in filtered tap water using Hyflo Super-cel as a filter aid (Table 8)

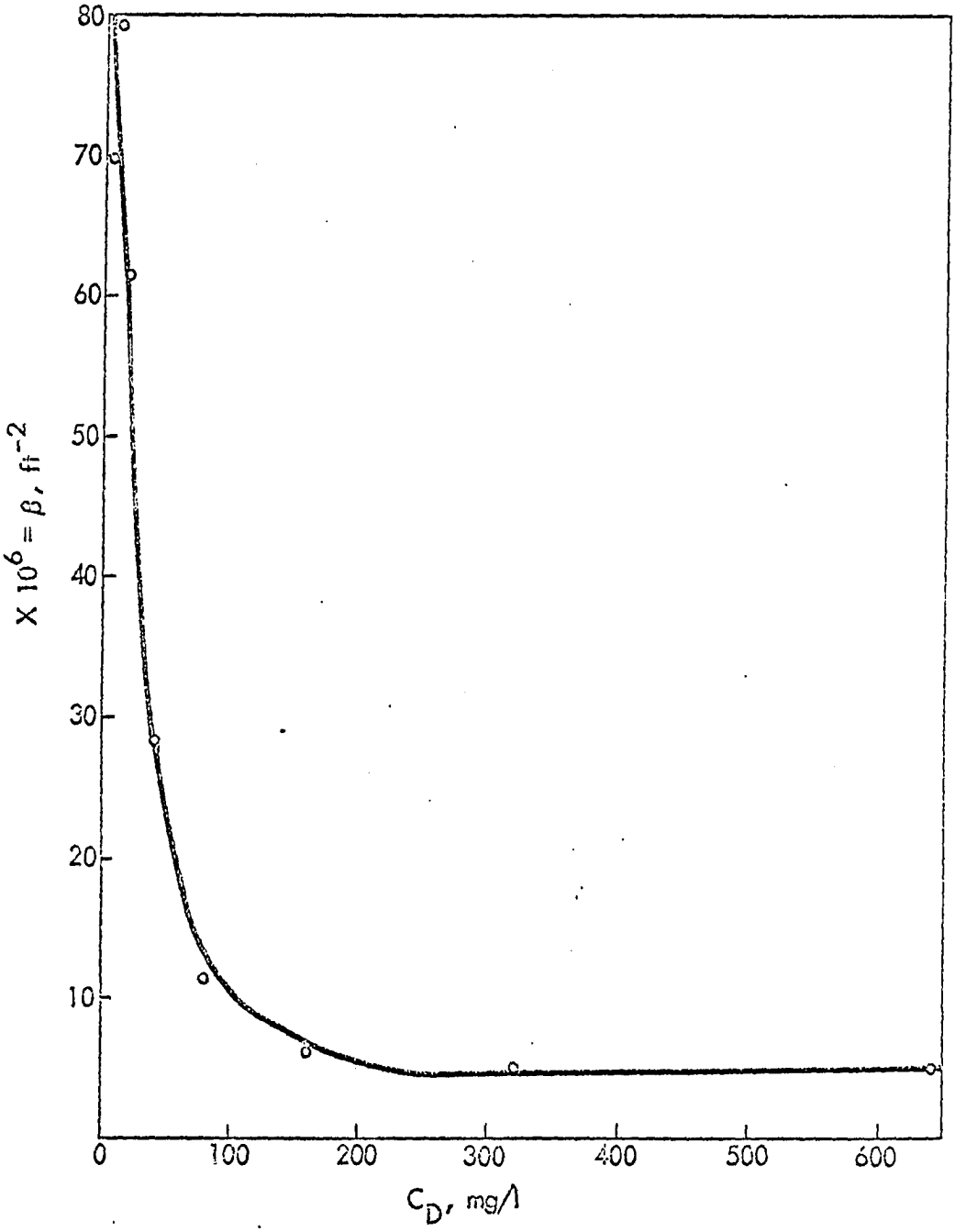

80
70
60
50
40
30
20
10
0
100
200
300
400
500
600
X 10^6 = β, ft^-2
C_D, mg/l

Figure 9. Variation of β-index with C_D at C_s/C_D = 0.37 in the filtration of Ball Clay in distilled water using Hyflo Super-cel as a filter aid (Table 9)

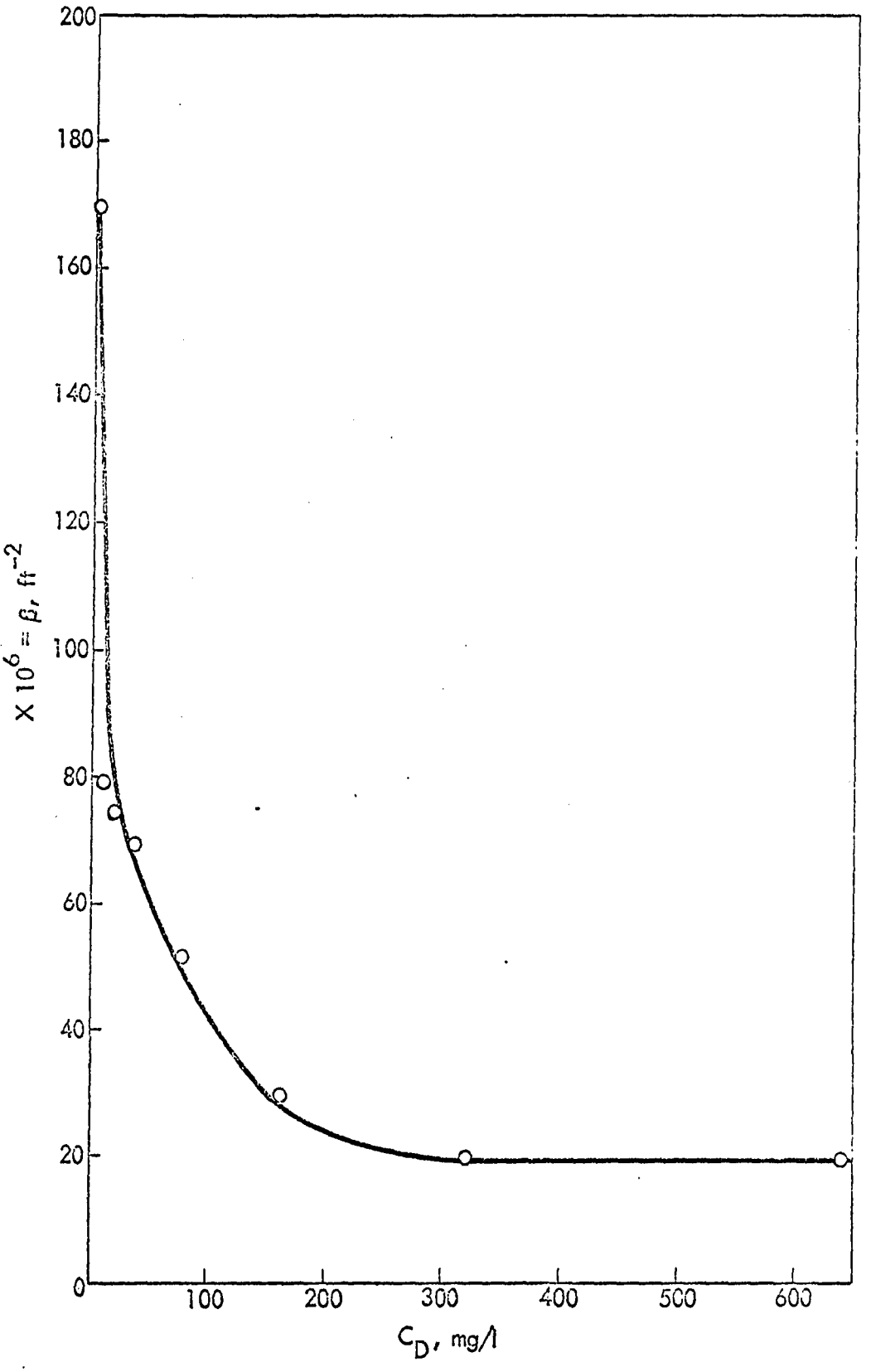
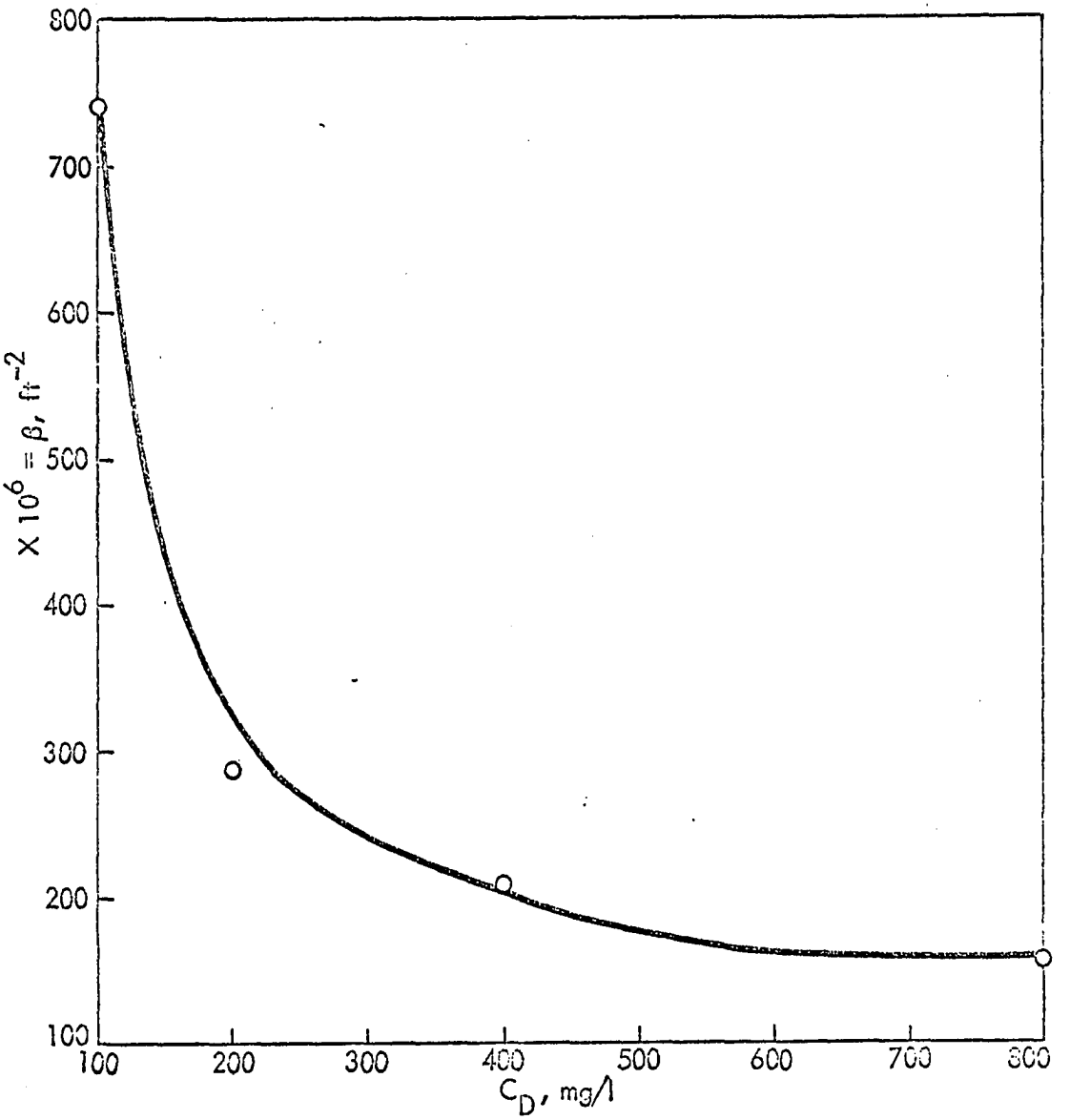

200
180
160
140
120
100
80
60
40
20
0
100
200
300
400
500
600
$\times 10^6 = \beta$, ft^{-2}
C_D, mg/l

Figure 10. Variation of β-index with C_D at $C_S/C_D = 0.457$ in the filtration of Black Hill Clay in filtered tap water using Hyflo Super-cel as a filter aid (Table 10)


800
700
600
500
400
300
200
100
X 10^6 = β, ft^-2
100
200
300
400
500
600
700
800
C_D, mg/l

by other researchers to explain the dependence of specific resistance on the concentration of solids have been discussed earlier in this thesis. For such a high dependence of specific resistance on solids concentration as observed in this study, some other explanation had to be found.

Heertjes (39, 59) performed constant pressure tests on dilute slurries and observed an exactly similar type of relationship between the specific resistance and the concentration of solids as shown in Figures 8-10. Heertjes reasoned that in the filtration of very dilute suspensions, the solids (the solids and the body feed in this case) are too small in quantity to form a reasonably thick cake during the filtration run. The only thing that happens in the filtration of such slurries is that the pores of the septum hardly get plugged during the run. According to the author, this is responsible for a high specific resistance since we are measuring the resistance of a partially clogged septum and not the resistance of a filter cake. As the solids concentration increases, the probability of formation of the cake also increases and the specific resistance so determined will represent the actual characteristics of the cake more closely. With a sufficiently high concentration of solids, enough cake is formed after a covering of the pores in the septum is accomplished that the specific resistance of the cake assumes a constant value.

A review of Figures 5 and 6 is suggested at this point. In the runs presented in Figure 5, it was explained that the cake was formed during filtration of the first 100 ml of suspension or even earlier resulting in a straight line relationship between $\Delta T/\Delta V$ and V from nearly the beginning of the run. In the runs of Figure 6, 350 ml of the suspension had to be filtered to provide a uniform cover of cake over the pores in the septum and to start an actual cake filtration due to the dilute concentration of suspensions used for these runs (C_s = 9.55 mg of settled Ball Clay in one liter of filtered tap water; C_D = 60 mg/l of Hyflo Super-cel). It is evident, therefore, that if the filtration of this suspension were stopped before the collection of the first 350 ml of filtrate, the value of the specific resistance found using the results of the run collected up to that time would have been about 250 percent larger (the ratio of slopes of the two straight lines). From this it follows that in runs made using suspensions which are even more dilute, such as those used in Runs 244, 245, 246, and 247, the entire suspension might have been used up in just providing a uniform covering of cake over the pores of the septum. This would be reflected in high values of specific resistance associated with these suspensions.

One way of evaluating how quickly a cake would be formed so that a test result would be meaningful, would be to

calculate the probable thickness of the cake which would be obtained from removal of all of the solids in a test sample. The following calculations showing the thickness of a cake in one of these runs may illustrate this point more clearly.

Calculations for the thickness of cake - Run 244

C_s = 1.94 mg/l

= 2 mg/l approximately

C_D = 5 mg/l

Assume bulk density of diatomite in the cake = 20 lbs/ft^3 or 321 $\frac{mg}{cc}$

Volume of suspension filtered = 900 ml.

Therefore, the weight of diatomite in the cake

= 5 mg/l x 0.9 l

= 4.5 mg

The volume of the cake, assuming that the solids do not increase the thickness of the cake = 4.5 mg/321 mg/cc

$= 1.4 \times 10^{-2}$ cc

The area of the cake = 0.891 sq in or 5.75 cm^2.

Therefore, the thickness of the cake = $\frac{1.4 \times 10^{-2}}{5.75} \frac{cc}{cm^2}$

$= 0.244 \times 10^{-2}$ cm

= 24.4 microns

The average diameter of a particle of Hyflo Super-cel is about 11-12 microns (8). Thus, even at the end of the run, the thickness of the cake is 24.4 microns, which is hardly more than two thicknesses of the diatom particles. After filtration of 100 ml of this suspension, when data collection during the run began, an average cake thickness of only 2-3 microns had built up, a thickness which is not even equal to the thickness of one diatom. If the solids buildup around the pores of the septum is also considered, a cake might be nonexistent even at the end of the run.

Al-Khafaji (50) used very dilute suspensions of clay in some of his runs and used cake filtration equations to calculate the specific resistance. Some of his results are undoubtedly erroneous because some of his suspensions were too dilute to form a cake in the early part of the run and some even at the end of the run. It appears that he did not fully appreciate the phenomenon of septum plugging and cake formation and its implications on the specific resistance thus determined. In his study he used pressures up to about 50 psi and such high pressures can only enhance the plugging (forcing the solids into the pores of the septum itself) of the septum.

Another explanation can probably also be given to explain the shape of the curves in Figures 8-10. When the slurry to

be filtered is very dilute, all the solids in the slurry are used to plug or cover over the pores of the septum, forming little or no cake. As a result of this, almost all the pressure loss is across the septum with a very small fraction, or no head loss at all, across the cake. It was stated earlier that in the computation of β-index from Equation 56 or 63, the pressure P should correspond to the pressure across the cake and not the total pressure. The total pressure can, however, be used when the pressure loss across the septum is negligible. In dealing with a dilute slurry, which causes only plugging of the septum throughout the run, the assumption of a negligible resistance of the septum is far from true. If the pressure across the cake only were to be used in the computation of β-index, the values of β-indices which look very high will be reduced considerably. From this explanation it also appears reasonable that the specific resistance may ultimately become almost constant when the concentration of solids is so high that the resistance of the built up cake far exceeds the resistance of the septum from the early part of the run to the end. In such a case, the assumption that the entire pressure loss occurs across the cake will be more true. It also follows that this assumption will be valid much earlier in a run when filtering a more resistant suspension than when filtering a suspension which possesses small

resistance.

The relation of the particle size in the slurry to the pore size in the septum might also be important. For a small value of this ratio, the possibility of septum plugging by penetration of solids into the septum is very high, a possibility which reduces as the particle size increases in relation to the pore size in the septum. It is also conceivable that the use of a high pressure in filtration is likely to cause the septum to be plugged more severely than the use of a low pressure. Carman (45) suggested that, in constant pressure filtration, the primary layer of the cake should be deposited at low pressure since otherwise the septum becomes plugged and the initial resistance becomes unduly high.

From the above reasoning it might appear that if a method could be devised to determine exactly the pressure loss across the cake and the value of this pressure were used in the calculations, the specific resistance would be independent of the solids concentration. This may not be true either. According to Heertjes (39) the orientation of particles in the slurry reaching the cake also plays an important part. In a dilute suspension each solid particle moves separately from the others and will easily follow the streamlines of flow directed towards the pores in the filter. The result will be that the particle will either enter a pore or will

cover the pore opening, depending upon its size as compared to the size of the pore. With increasing concentration, more particles will arrive near the pores at the same time. This may not only decrease the chances of blocking because of the large number of particles competing to get entry through the pores, but it may also provide less opportunity due to mutual interference for the particles to orient themselves to give a dense packing. Highly concentrated slurries may also permit flocculation of particles themselves, which may also be responsible for decreasing the specific resistance. Exactly an opposite situation will prevail in a dilute suspension.

In making specific resistance tests using constant pressure techniques, we might encounter three distinct solid concentration problems:

a) The solids concentration might be so small that a cake never forms during the test, so cake resistance is never measured.

b) The solids concentration might be so high that the mutual interference between particles might cause significant variation in cake porosity or mutual flocculation of solid particles.

c) The solids concentration might be in a range between these two extremes and, then, it would be impossible

to determine when the resistance of the septum is negligible compared to the resistance of the cake.

Ruth (36) explained the dependence of specific resistance on solids concentration on the basis of electro-osmosis. Electro-osmosis in relation to filtration has already been discussed briefly. It appears from this study that filtration resistance variations may be explained partially on the basis of electrokinetics. The settled Ball Clay suspensions in distilled water were found to be three to four times more resistant to filtering than identical suspensions in filtered tap water. See results in Tables 8 and 9. This observation may be qualitatively explained on the basis of electrokinetics; higher ionic concentration in the tap water producing a thinner double ionic layer resulting in the development of a smaller magnitude of electro-osmotic pressure. A similar observation was also reported by Bishop et al. (40).

From the discussion of "septum plugging preceding cake formation" it might appear that if the run is continued for a long time so that a cake is formed, the results of the constant pressure runs should agree reasonably well with the results obtained in the runs on the constant rate pilot plant. However, the runs made on iron bearing waters, using body feed concentrations as high as 800 mg/l when cake formation was certain even in the filtration of the first 100 ml of

the suspension, disproved this. See results in Tables 6 and 7. Some runs, which will be reported later (Table 14), were made on the modified constant pressure apparatus (Figure 4) and were continued for a long time to insure the formation of a cake. Even the results of these runs did not agree satisfactorily with the results of the constant rate runs.

To evaluate the possible reasons for this disagreement, it is pertinent to study the formation and the porosity distribution within a cake. As liquid flows frictionally through a bed of compressible solids (all solids are compressible to some extent), viscous drag on the particles produces an accumulative compressive pressure which causes the porosity to decrease as the septum is approached. For point contact between solids, the following relation holds (60):

$$d P_s + d P_x = 0 \qquad (64)$$

where

P_x = the hydraulic pressure at a distance x from the septum (Figure 11)

P_s = the solid compressive pressure.

Integration of the above equation for constant pressure filtration yields

$$P_s + P_x = P \qquad (65)$$

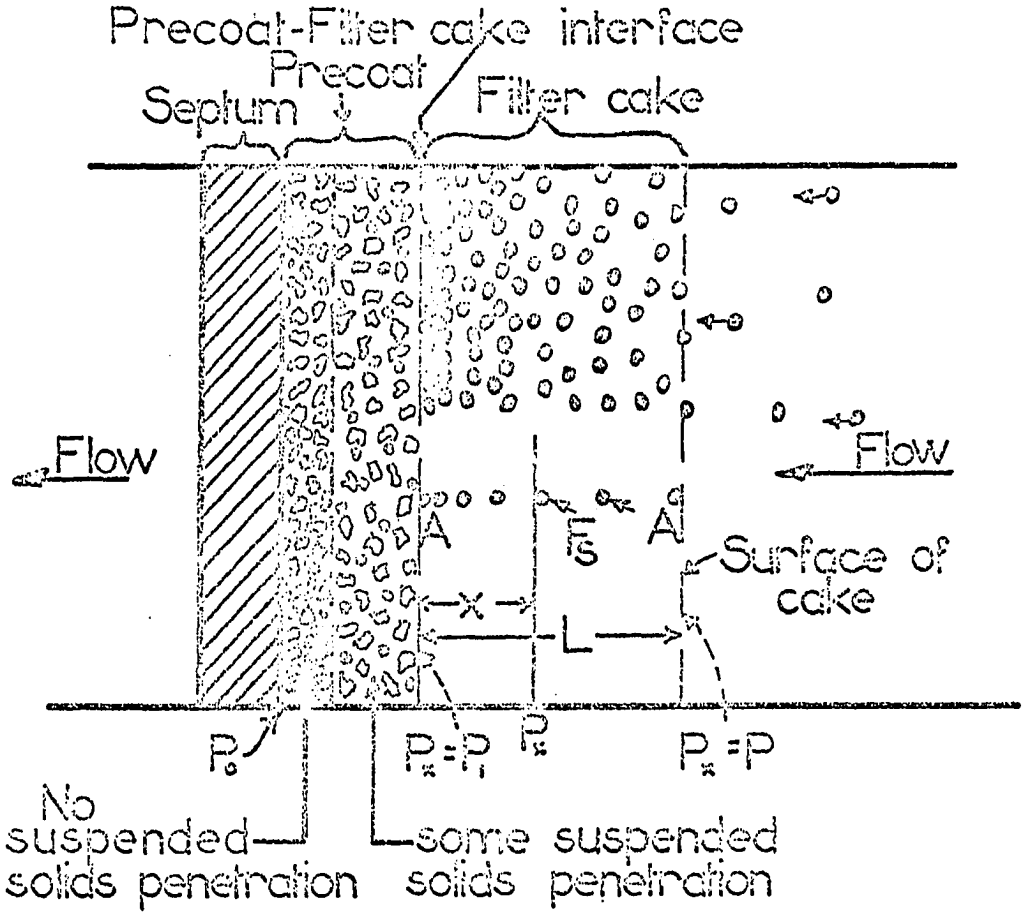
where

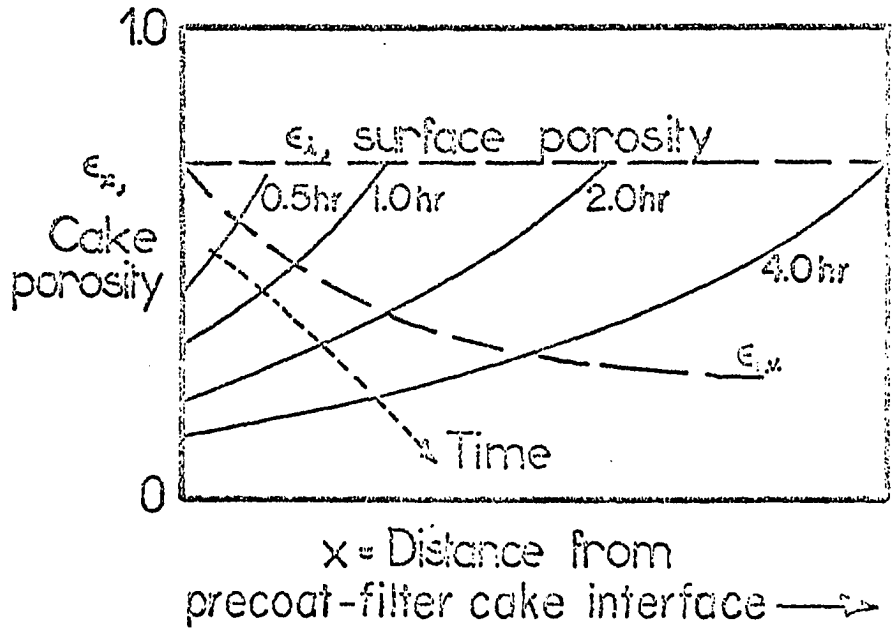
P = the applied pressure at the surface of the cake.

Thus, the maximum solid compressive pressure, P_s, will be near the septum where the liquid pressure P_x in the cake will be the least. Similarly the liquid pressure P_x will be the maximum at the surface of the cake with P_s being zero at that point. Since it is the solid compressive pressure that causes compression of the cake and a decrease in porosity, the cake will be least porous near the septum and the most porous at the surface. In other words, there will be a porosity gradient within the cake at any particular instant during a filtration run. As time passes the cake thickness increases, and at a given value of x, the porosity decreases. The porosity at the surface of the cake, however, always remains nearly constant throughout the run. Thus, there is not only a spatial variation of porosity within a cake, but a time variation as well. See Figure 12 (61). Since the specific filter cake resistance depends on cake porosity (Equation 46), it is not constant within a cake and this variation changes with filtration time. In fact, the specific resistance defined so far by different equations is only an average value, which was defined by Ruth (36) as follows:

Figure 11. Section through filter cake and medium showing porosity gradient because of frictional drag

Figure 12. Spatial and time variation of porosity within a filter cake


Precoat-Filter cake interface
Precoat
Septum
Filter cake
Flow
Flow
A
A'
Surface of cake
x
L
P_0
$P_x = P_1$
P_x
$P_x = P$
No suspended solids penetration
some suspended solids penetration


1.0
ϵ_x, surface porosity
ϵ_x, Cake porosity
0.5hr
1.0hr
2.0hr
4.0hr
ϵ_M
Time
0
x = Distance from precoat-filter cake interface

$$\alpha = \frac{P - P_1}{\int_0^{P - P_1} \frac{d P_s}{\alpha_x}}$$

where

P = total pressure drop across the filter (septum, medium and cake)

P_1 = pressure required to overcome the septum and medium resistance

α_x = point specific cake resistance.

In constant rate filtration also a similar variation of cake porosity, both in time and space, exists within the cake. Filtration at constant rate can be regarded as a series of infinitely small periods of filtration at constant pressure but with steadily increasing pressures. It is possible, therefore, that the pattern of porosity distribution within a cake laid during constant rate filtration may be different than within a cake laid in constant pressure filtration even under supposedly identical conditions. If this is true, the average specific resistance in both filtration processes may also be significantly different. The variation in porosity shown in Figures 11 and 12 is only qualitative and perhaps exaggerated. The actual variation will depend on the nature of the slurry, the type and concentration of the filter aid used, the pressure used in filtration and other factors. In an incompressible cake formed by filtering a raw water

to which reasonable quantities of a filter aid are added as in potable water precoat filtration practice, such variations in porosity might be less. Heertjes (39, 59) observed that the specific cake resistance is also affected by the filtration velocity, increasing as this velocity decreases. It is therefore possible that the different velocity patterns in the constant rate and constant pressure filtration runs (where as the filtration velocity is constant in the first process; it keeps on decreasing in the second process) might also be responsible for the disagreement between the results of identical runs made in the two processes. Another factor responsible for unsatisfactory correlation of results of the two processes may be the different septum conditions. In a constant pressure filtration run, the full pressure is effective throughout the run and may cause a severe plugging of the septum, especially at high pressures of filtration.

Carman (45) observed that the total resistance of a dirty septum at the end of a constant pressure filtration run was ten times larger than the total resistance of the same septum observed in the beginning of the run. This problem may be less severe in a constant rate filtration operation where the pressure increases gradually. In this case a sufficiently thick cake would have already built up before the pressure is high enough to cause plugging of the septum. Thus, the septum is relatively more protected. In

the former process, where the septum plugging is expected to be significant, the assumption of a negligible pressure loss across the septum may be less true. The amount of error by making this assumption is likely to decrease with more resistant suspensions, where the resistance of the cake may build up very fast. That might be one of the explanations of the observation that agreement between the results of constant pressure filtration and constant rate filtration improved with increasing C_s/C_D (Tables 6 and 7).

Attempts to predict the results of constant rate filtration on the basis of the results obtained in constant pressure filtration and vice versa are not recent. Walas (62) measured porosity of highly concentrated chemical slurries cakes laid in constant rate filtration and constant pressure filtration under identical conditions and found a good agreement of the porosity in the two cases. He stated that good correlation is possible if the terminal pressure reached in constant rate filtration is the same as the pressure used in constant pressure filtration. He concluded that the porosity of a filter cake does not depend seriously on how it is formed, but only on the final pressure to which it is subjected. In other words, it implies that the effect of a particular pressure on porosity, whether applied for a long time (as in constant pressure filtration) or for an infinitesimal length of time (as in constant rate filtration), is

the same. This does not appear to be rational. Tiller[1] reported that all the resistances calculated by Walas (62) were erroneous and corrected values were published by the author in the literature at a later date. Tiller[1] appears to have serious reservations about the data collected by Walas (62).

Tiller further feels that a good correlation between the constant pressure and constant rate filtration results can only be obtained in highly specialized cases. One of the problems stated by Tiller in the correlation of constant pressure and constant rate filtration results is the side-wall friction effect, which may be serious enough to distort the uniformity of the cake and the stress distribution. This may affect the flow patterns of the liquid within a cake. It cannot be said with certainty on the basis of the current knowledge as to what ratio of cake thickness to cell diameter will be necessary to eliminate this effect. The error due to this effect may be different in the two apparatuses. Another possible source of error pointed out by Tiller is based on his belief that the specific resistance of a moving bed[2] is just as likely to change as that of a fixed bed, the

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[1]Tiller, F. M. University of Houston, Texas. Filtration resistance. Private communication. 1969.

[2]Moving bed is one in which the bed continues to build up during filtration, e.g. a filter cake.

latter having been frequently reported in literature (36, 42, 43).

Walker et al. (46) predicted the expected resistance of some chemical slurries in constant pressure filtration from the results obtained in constant rate filtration and found the prediction in reasonable (?) agreement with the experimental results (± 33 percent). The data reported, however, is fragmentary.

In the opinion of Carman (45) the discrepancies between the results of constant rate and constant pressure filtration are not real, but are due to 1) experimental difficulties in maintaining a constant rate or a constant pressure, 2) use of large-scale apparatus in constant rate tests and consequent difficulty in insuring accurately controlled conditions, and 3) difficulty of analyzing the data so as to make proper corrections for septum resistance.

For constant pressure filtration equations to be applicable, the head loss across the septum must be negligible. If there is significant penetration of the solids into the septum or into the septum plus its precoat, it becomes exceedingly difficult to evaluate the cake thickness required to make the resistance through the septum negligible. The implication of this has been discussed before in this thesis. The amount of slurry which will have to be filtered before the assumption of a negligible septum resistance can be justified will

depend on the type of septum, the type of slurry, the pressure used and several other factors. The continued filtration, however, done in some of the runs made on the modified apparatus (Figure 4) was expected to achieve this objective; still it did not improve the correlation between the constant pressure and constant rate results (Table 14).

Need for Building a Small-Scale, Constant-Rate Apparatus

From the results presented thus far and on the basis of the theoretical considerations discussed, it seems apparent that the results of a constant pressure filtration may not predict the performance of constant rate filtration successfully. The problem does not seem to be in the inadequate understanding of the two processes, but in the lack of availability of a sound and complete theory that can quantitatively account for all the factors involved in filtration. Admittedly, the understanding of the phenomenon of filtration is far from complete.

For example, it was explained that the variation of porosity within a cake undoubtedly will be different in the two processes. Even small differences in porosity can significantly affect cake resistance (Table 1). In spite of a qualitative understanding of the variation and this difference, no theory is available to account for these differences.

There is an awareness of the problem of septum plugging and cake formation but it seems impossible to account for this in a simple mathematical way. These problems have been discussed in detail and no purpose will be served by listing them again.

It was realized, therefore, that only a small scale apparatus designed to be used for filtration at constant rate can predict the results of filter operation in the field at constant rate. The designed apparatus should be equipped to control the flow rate, and its complete operation should include all the steps involved in actual field scale filtration: precoating, filtration, and backwashing.

The usual requisites of any practicable laboratory scale apparatus were also considered essential:

1) simple operation
2) inexpensive to build or construct
3) small space requirement
4) easy to carry from place to place
5) small test water volume and filter aid weight requirements

In short, it seemed evident that the apparatus should simulate the field conditions as closely as possible and still be free from the complexities of large scale field equipment or even of laboratory-scale pilot plant equipment.

With the above considerations in view, an apparatus was designed and built. It is described in the next chapter.

SMALL SCALE CONSTANT RATE FILTRATION APPARATUS

Description of Apparatus

This apparatus will be called the SSCR filter (small scale constant rate or SSCR filter). A schematic diagram of this apparatus is shown in Figure 13. Figure 14 is a photograph of the apparatus. This apparatus consists of a one liter capacity raw water tank, a precoat pot also of one liter capacity, a flat septum filter about two inches in diameter (area = 3.14 inch^2) with arrangement to regulate and observe the flow rate through a rotameter[1] and to measure the head loss across the filter cake using a 64 cms mercury manometer. The filter assembly was built of plexiglass to permit viewing of the cake during the run. The precoat pot and the raw water tank were also made of plexiglass due to the fact that they were readily available in the laboratory as surplus sample holders from the constant pressure filtration apparatus. Appendix D contains the working drawings of the filter. Similar drawings for the precoat pot and the raw water tank are presented elsewhere (54). A centrifugal pump driven by an electric motor was used both to withdraw water from the backwash tank (which was the same as the raw water tank) during the backwashing operation and to pump

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[1]Model No. 2-1355-V, SHO-RATE, a patent of Brooks Instrument Company, Inc., Hatfield, Pa.

Figure 13. Schematic diagram of SSCR filter

a. indicates flow path during precoating

b. indicates flow path during filtration

c. indicates flow path during backwashing

} shown by thick lines

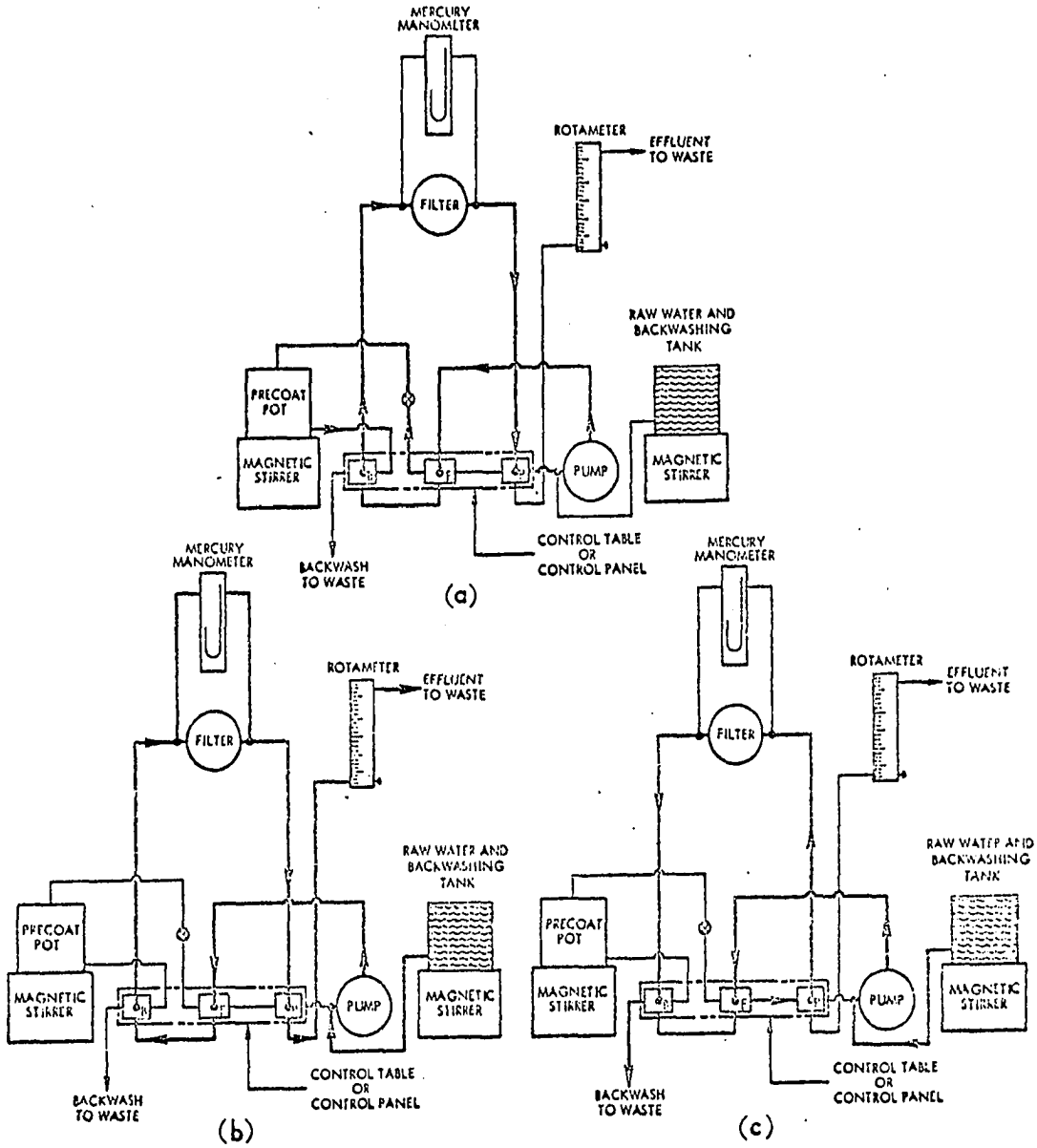
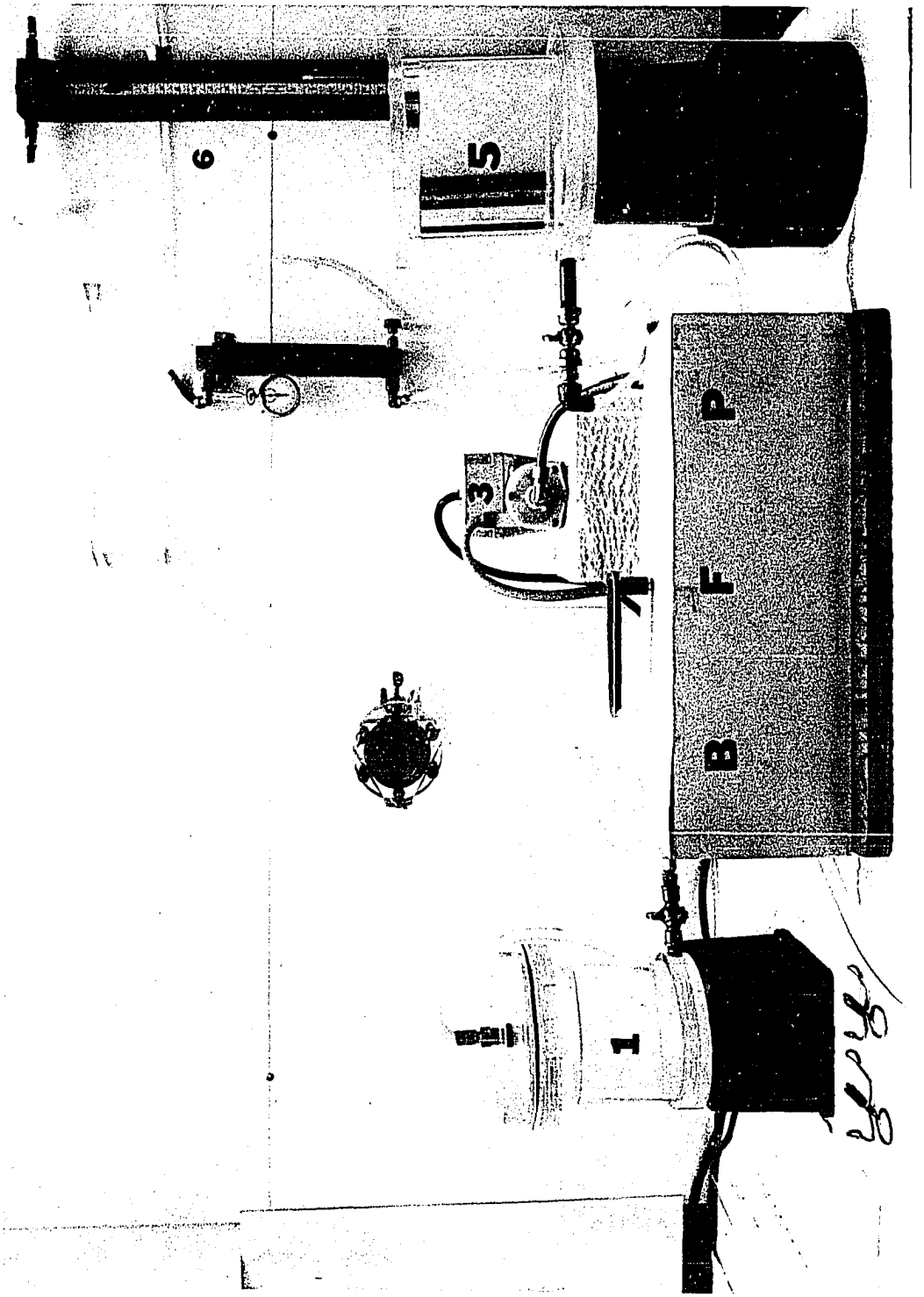

MERCURY MANOMETER
ROTAMETER
EFFLUENT TO WASTE
FILTER
RAW WATER AND BACKWASHING TANK
PRECOAT POT
MAGNETIC STIRRER
PUMP
MAGNETIC STIRRER
BACKWASH TO WASTE
CONTROL TABLE OR CONTROL PANEL
(a)
MERCURY MANOMETER
ROTAMETER
EFFLUENT TO WASTE
FILTER
RAW WATER AND BACKWASHING TANK
PRECOAT POT
MAGNETIC STIRRER
PUMP
MAGNETIC STIRRER
BACKWASH TO WASTE
CONTROL TABLE OR CONTROL PANEL
(b)
MERCURY MANOMETER
ROTAMETER
EFFLUENT TO WASTE
FILTER
RAW WATER AND BACKWASHING TANK
PRECOAT POT
MAGNETIC STIRRER
PUMP
MAGNETIC STIRRER
BACKWASH TO WASTE
CONTROL TABLE OR CONTROL PANEL
(c)

Figure 14. Photograph of SSCR filter

1 = Precoat pot
2 = Filter
3 = Pump
4 = Rotameter
5 = Raw water tank and Backwashing tank
6 = Manometer
B = The position of the lever during backwashing
F = The position of the lever during filtration
P = The position of the lever during precoating


6
5
3
P
F
B
1

raw water through the filter during the filtration operation. The precoat pot and the raw water tank were mounted over magnetic stirrers to prevent settling or segregation of their contents.

All connections between the pump, the precoat pot, the raw water tank, and the manometers, etc. were made with 1/4 inch I.D. tygon tubing. A small lever provided on the top of a 15 inches x 3 inches control panel could be moved to any of three positions, each position corresponding to a particular operation in the filtration cycle, i.e., precoating, filtration and backwashing. The movement of the lever simultaneously actuated a series of three gears, which in turn opened and closed the appropriate valves so as to change the direction of flow.

Procedure

The sequence of filtering operations was exactly the same as in the pilot plant or in field filter operation. A weighed amount of precoat filter aid (0.15 lb per ft^2 of flat septum) was added to the precoat pot containing one liter of distilled water. Mixing was provided using a 2-in. magnetic bar and the magnetic stirrer. The pump[1] was started and the lever on

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[1]Pump Model No. 2, E-38N, Patent No. 194, 570. Little Giant Pump Co., Oklahoma City, Oklahoma.

the control table was moved to position P. The filter aid slurry was circulated through the filter until all of the filter aid was deposited and the distilled water was clear.

The raw water tank was filled with one liter of the water to be filtered and a weighed amount ($\pm$ 0.0002 g) of body feed was added. The control panel lever was moved to position F which started the actual filtration. The flow rate was adjusted and held constant using the rotameter. During the filtration run the following data were observed and recorded at appropriate intervals:

1) head loss
2) clock time.

The effluent quality was monitored using the analytical techniques explained earlier. Samples of the effluent were collected over a period of 10 minutes at least once per run during most of the runs. The time and volume of sample collected were recorded to insure that the flow rate was steady and agreed with the rotameter calibration curve. The rotameter was calibrated in the laboratory.

For a filtration rate of about 1 gpm/ft^2 (about 86 ml/min), one liter of the raw water sample lasted about 10 minutes. More raw water and the body feed were added to the raw water tank after every 10 minutes as long as needed.

Addition of body feed filter aid to a new 1-liter raw water sample at about 10-minute intervals served to

provide a more precise and more uniform body feed during cake formation. Prior to the start of a run, sufficient body feed additions were weighed to last for the estimated length of run.

After filtration was completed, which was determined either from consideration of the maximum capacity of the pump (about 26 cms Hg at a flow rate of 1 gpm/ft^2) or when a well-defined curve of head loss versus time was obtained, the control panel lever was moved to position B. The filter cake was automatically removed with a swirling motion of water and was drained to waste. Distilled water was continually added to the raw water tank till the cake was washed off the septum and the system was clean.

PRESENTATION OF SSCR FILTRATION RESULTS

General

After designing and building the new apparatus based on the principle of constant rate filtration, it was necessary to put it to test for reproducibility of results and comparison with the results obtained on the pilot plant. Then the apparatus could be used to study the effect of concentration of solids on the specific filter cake resistance.

Reproducibility

Reproducibility was checked several times on different types of waters used during the course of this study. It was found that the runs made on this apparatus under exactly identical conditions produced reproducible results ($\pm$ five percent). A summary of three such runs made on unsettled Ball Clay in tap water, using Hyflo Super-Cel as a filter aid, is presented in Table 11. The head loss vs time of filtration data collected in these runs are plotted in Figure 15.

Equation 4 (Figure 2) was used to calculate the values of β-indices listed in the last column of Table 11. Appendix B contains an example showing the calculations of β-index for a typical run.

Runs on Iron Bearing Water

In an earlier study (3) several hundred runs were made on the pilot plant using iron bearing waters (7-8 mg/l Fe^{+++}) for filtration. It was therefore decided to filter the same water through the new SSCR filter and compare the results obtained with the earlier results. A good agreement would provide conclusive proof of the successful use of the apparatus.

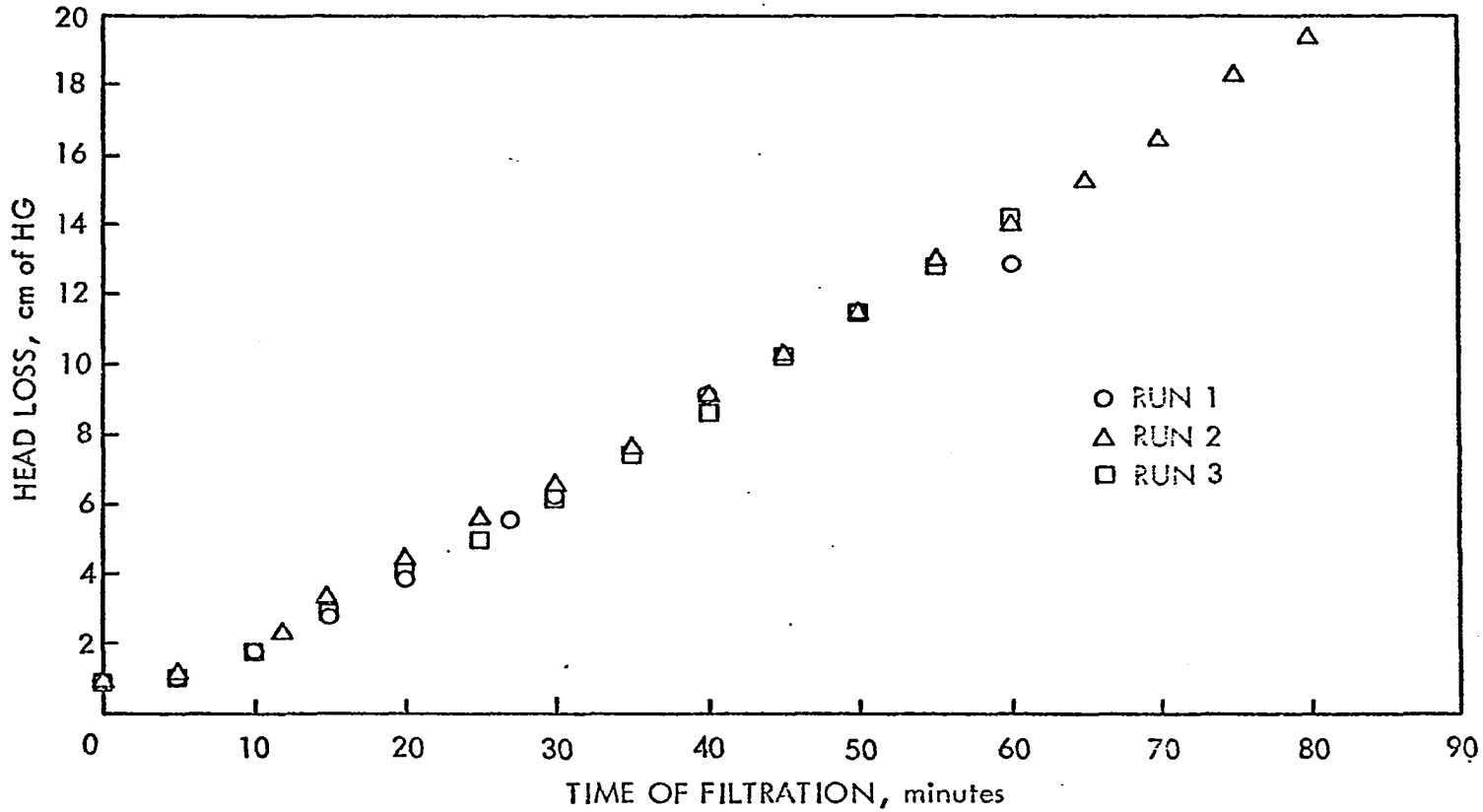
Table 11. Summary of three identical runs to demonstrate reproducibility of data collected using the SSCR filter

Run no.	C_s mg/l	C_D mg/l	C_s/C_D	$\beta/10^6$ ft^{-2}
1	71	90	0.79	11.6
2	71	90	0.79	11.5
3	71	90	0.79	12.0
			Mean	11.7
			Max Variation, %, from Mean	2.6

Condition of runs

Filtration rate, q = 1.05 gallons per minute per ft^2
Average temperature = 24° C = 25° C
Precoat = 0.15 lb/ft^2
Filter aid used = Hyflo Super-Cel
Suspension used = Unsettled Ball Clay in unfiltered tap water

Figure 15. Runs to demonstrate reproducibility of identical runs using the SSCR filter (see Table 11)


HEAD LOSS, cm of HG
20
18
16
14
12
10
8
6
4
2
0
10
20
30
40
50
60
70
80
90
TIME OF FILTRATION, minutes
RUN 1
RUN 2
RUN 3

Comparison of SSCR and pilot plant results

Two series of runs were conducted in the filtration of iron bearing waters 1) using C535-X as filter aid and 2) using S2-3[1] as filter aid. In calculating the expected specific resistance of filter cakes in the pilot plant, with which the results obtained on the SSCR filter were compared, the equation for β listed in Table 2 was used for C535-X. Another equation was developed earlier (3) for S2-3 and was, therefore, used for this filter aid.

$$\beta = 10^{a} (C_s/C_D)^{b}$$

where

a = 10.4503

b = 2.2428

The data collected in the runs made on the SSCR filter were used to calculate the specific filter cake resistance, β-index, for each run using manual calculations (similar to a sample calculation shown in Appendix B). A summary of these results is contained in Table 12. The expected results based on pilot plant runs under identical conditions are also summarized in this table. In all these runs, the filtration

[1]Sample of a perlite taken from bag No. 3 of Sil-Flo grade 272, a product of Sil-Flo Corporation, Fort Worth, Texas (3).

Table 12. Comparison of SSCR filtration results with the expected results on the pilot plant on iron bearing water

Run no.	C_s mg/l	C_D mg/l	C_s/C_D	$\frac{\beta}{10^6}$ ft^{-2} [a]	$\frac{\beta_p}{10^6}$ ft^{-2} [b]	$\frac{(\beta-\beta_p)}{\beta_p} \cdot 100$	Remarks
21	7.85	400	0.0196	5.43	8.00	-32	1) Iron concentration in effluent in all these runs was less than 0.06 mg/l.
23	8.27	400	0.0207	7.1	8.77	-19	
24	8.26	320	0.0258	9.9	13.71	-28	2) Runs 21-27 and Runs 34-35 were made using C535-X
25	8.26	266	0.0310	19.7	19.45	+1.3	
26	8.26	230	0.0359	25.6	25.88	-1.1	3) Runs 28-33 were made using S2-3
27	8.36	400	0.0209	8.95	9.057	-1.2	
28	8.16	400	0.0204	3.54	4.52	-22	4) Average temperage[c] ranged from 23°C-26°C.
29	8.16	266	0.0307	8.70	11.23	-22.5	
31	8.35	320	0.0261	5.97	7.85	-24	5) In all runs the septum was precoated except in Run 27, where a 0.45 micron pore size millipore filter membrane was substituted in place of the precoat.
32	8.36	200	0.0417	18.5	22.59	-18.1	
33	7.57	320	0.0237	5.8	6.31	-8.1	

[a] These values are for the SSCR filter runs.

[b] β_p = β expected from the pilot plant.

[c] The average of the influent and the effluent temperatures was taken in each run and used for the calculation of β-index.

Table 12 (Continued)

Run no.	C_s mg/l	C_D mg/l	C_s/C_D	$\frac{\beta}{10^6}$ ft^{-2} [a]	$\frac{\beta_p}{10^6}$ ft^{-2} [b]	$\frac{(\beta-\beta_p)}{\beta_p} \cdot 100$	Remarks
34	7.57	266	0.0285	16.9	16.71	+1.2	
35	7.73	400	0.0193	6.5	7.73	-16	

rate was held constant at 1.05 gpm/ft^2.[1]

The results in Table 12 indicate that the SSCR filter predicted the pilot plant performance relatively closely. The agreement between the results of the pilot plant and those obtained on this new apparatus was satisfactory. In a few cases the difference was more than 20 percent, 32 percent being the maximum in one run. In about half of the runs, the agreement was within less than 10 percent. In a practical situation, where the prediction will be based generally on the basis of at least 2-3 identical runs to represent a particular C_s/C_D ratio, the percent difference between the predicted and an actual performance will be reduced. It might also be mentioned here that about 10 percent difference between the results of the pilot plant and those of the SSCR filter may be accounted for to the possibility that the filtration rate in the pilot plant might have been about 1.05 gpm/ft^2 instead of a value of 1.0 gpm/ft^2 assumed for the calculation of β-indices. A fresh look at the pilot plant rotameter calibration curve, which was

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[1]Actually the runs were planned to be made at a filtration rate of 1 gpm/ft^2. The rotameter was calibrated and adjusted to permit this filtration rate. After several runs had been made a 10 minute sample of the filtrate was collected, which indicated that the actual filtration rate was 1.05 gpm/ft^2. In all the subsequent runs, therefore the filtration rate was continued at 1.05 gpm/ft^2. It was made a regular practice to collect at least two effluent samples in each run and to insure this filtration rate. The calculations of β made in earlier runs were corrected to this filtration rate.

prepared about two years ago, strongly suggests such a possibility.

In the inlet and outlet connections to the SSCR filter pump, which were made of 3/8 inch I.D. tygon tubing, some settlement of the solids, both iron and body feed, was noticed. Frequently the tubing had to be pinched and shaken to carry these solids into the main fluid stream. After making 39 runs this tubing was replaced by 1/4 inch I.D. tygon tubing which nearly eliminated this problem. This might also be contributory to a small extent to the difference between the results on the two apparatuses.

In the calculations of β-indices on the pilot plant runs, the influent temperature (16° C) was used to determine viscosity. It was not recognized at that time that the temperature of the filter effluent can be significantly different than that of the influent. The correct procedure would be to use the average effluent temperature in such calculations. In the runs made on SSCR filter it was observed that the effluent was generally at a higher temperature by about 5°-6° C so that the average temperature was about 3° C higher than that of the influent. The use of the average effluent temperature in the calculations of pilot plant results instead of the influent temperature (19 °C instead of 16° C) would have lowered the values of β-indices by at least eight percent. These corrected values would be

closer to the values obtained on the SSCR filter. In spite of these possible sources of error, the percent difference between the results of the runs made on both the apparatuses is not significant where least cost filter design is involved.

POPO, the computer program for least cost design assumes a β value equal to 100 percent for the primary calculations. Then, POPO automatically assumes that the β is too low by 25 and 50 percent and too high by 25, 50 and 75 percent and recalculates the filtration costs and design combinations. The engineer can then decide the probable effects of errors in β on filter design. Typical POPO runs indicate that an error of 25 percent in β will not significantly affect overall filtration costs or the optimum filter design combinations.

In addition to the filter cake resistance the SSCR filter also successfully predicted the quality of the filtered water. The effluents in the runs made on both apparatuses had iron concentrations less than 0.06 mg/l.

It is therefore felt that the new filter can be used to predict both filtered water quality and specific filter cake resistance.

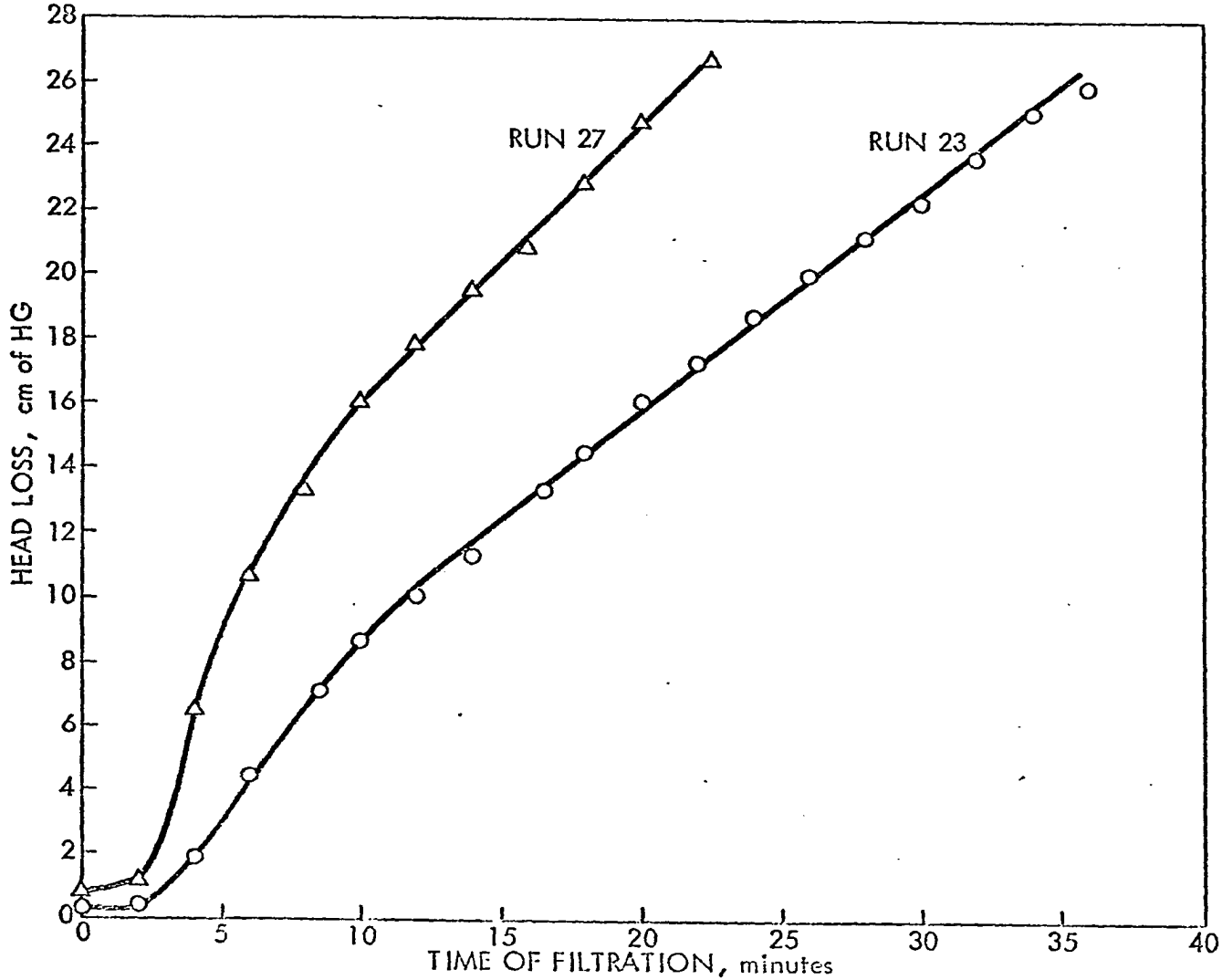
Septum plugging and cake formation

The phenomenon of septum plugging and cake formation was discussed earlier in this thesis, while presenting the results of the constant pressure apparatus runs. It was explained that it might take the filtration of several hundred milliliters of suspension to cover the septum uniformly and provide a uniform formation of cake depending upon the concentration of body feed and the suspended solids in the suspension, the pressure used in filtration, the nature of the septum and the nature of the filter aid and the solids. It was recognized that a larger volume of a dilute suspension might be needed to cover the openings of a septum than of a concentrated suspension of the same suspended solid. See Figures 5 and 6.

It was indicated in Table 12 that the septum was precoated in all the runs except in Run 27, where a millipore membrane with pore size equal to 0.45 micron was substituted in place of the precoat. The results of this run along with the results of a similar run using a precoat are shown graphically in Figure 16. It is evident from the results plotted in Figure 16 that about 12 to 15 minutes of filtration had to take place before the septum coverage had occurred and uniform cake formation started. After this time the head loss started increasing linearly with time, the rate of increase being a function of the nature of

Figure 16. Time of filtration versus head loss in the filtration of iron bearing waters

	C_s	C_D	Septum
Run 23	8.27 mg/l	400 mg/l	Precoat of C535-X
Run 27	8.36 mg/l	400 mg/l	Millipore membrane


RUN 27
RUN 23
HEAD LOSS, cm of HG
TIME OF FILTRATION, minutes
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
0
5
10
15
20
25
30
35
40

the suspension being filtered and the characteristics and amount of the filter aid used. Since both runs shown in Figure 16 were made under almost identical conditions, the rates of head loss increase in both these runs after the formation of the cake were nearly equal within the tolerances attributed to normal experimental errors. Similar runs on clay waters, which will be reported later, confirmed this conclusion. The results shown in Figure 16 demonstrate very clearly the different plugging patterns of the precoat and the millipore filter membrane. The plugging was faster in the membrane as indicated by a sharp increase of head loss in the initial filtration period. Due to this the run using the membrane had to be terminated earlier than the run with the precoat. In the membrane septum run, a head loss of 26.7 cms Hg was reached in 22.5 minutes, whereas it took 36 minutes for an equal head loss to build in the precoat run. This also shows that the values of the specific resistance calculated on the basis of the earlier part of the runs, if these were terminated before the formation of the cake, would not only have been erroneous but also different in both runs.

Septum plugging will be discussed in greater detail when the results of the runs made on clay bearing waters are presented.

Dependence of specific resistance on the concentration of solids

In order to study the effect of the concentration of solids on the specific resistance of the filter cake, runs were conducted on iron bearing waters using varying concentrations of the suspended solids, i.e., C_s. The amount of the copper sulfate added to the raw water was also proportionately changed. In the absence of an effect of concentration of solids on specific resistance, the value of the β-index will be the same for any particular value of C_s/C_D. The results of this study indicated that this was not true for this water. These results are summarized in Table 13 except the results of the runs with a C_s value in the range of 7.5-8.5 mg/l, which are presented in Table 12. Figure 17 shows the results graphically.

It is evident that the β-index depends on the concentration of solids, although the pattern of variation does not follow a systematic trend. Earlier studies in the field of chemical engineering (36, 38) almost always indicated that the specific resistance decreased with increasing concentration of solids. This rule does not always seem to be valid for runs made at a C_s value of 2.0 mg/l, the line for which intercepts the corresponding line drawn for the runs made at a C_s of 4.0 mg/l. If the data collected by LaFrenz (26) is analyzed and represented by a plot similar to Figure 17,

an identical relationship is found to exist between the runs made at the same three iron concentrations (i.e., 2, 4, 8 mg/l). There does not appear to be a significant difference between the specific resistance observed for an iron concentration of 8 mg/l and 13 mg/l for a particular value of C_s/C_D, although data collected at a C_s of 13 mg/l are inadequate.

The various possible reasons for the dependence of resistance on solids concentration have been stated earlier. It is possible that the floc characteristics of waters containing different concentrations of iron might be different, which may be partly responsible for the above observation, in addition to the factors already discussed. Also, the concentration of both solids and filter aid may affect the adsorption-straining relationship in determining how and where the solids are removed in the cake. Whatever be the reasons, it appears that the assumption of the independence of resistance on the concentration of solids in the design of a practical installation might be dangerous.

Runs on Clay Bearing Water

Work on the SSCR filter was started using clay bearing waters for filtration for no definite reasons. Only after a few runs had been made and some problems investigated were the runs on iron bearing waters started, the results of which

Table 13. Results of runs using varying concentrations of iron as suspended solids and C535-X as filter aid

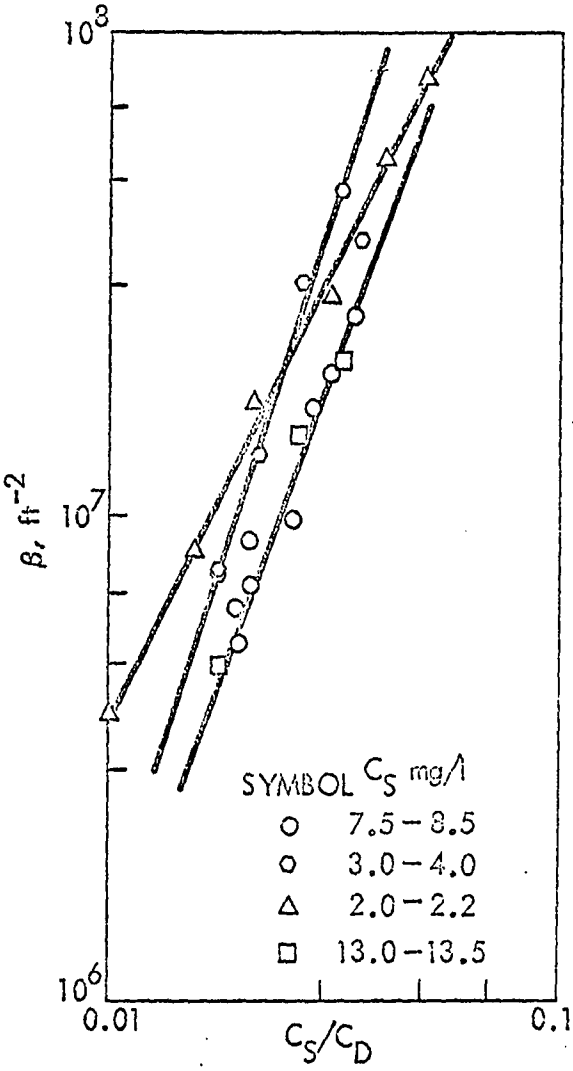
Run no.	C_s mg/l	C_D mg/l	C_s/C_D	$\beta/10^6$	Remarks
36	3.54	200	0.0177	7.75	1) Iron concentration in effluent in all runs was less than 0.06 mg/l.
37	3.36	100	0.0336	46.5	
38	3.59	133	0.027	30.0	2) Average water temperature in these runs ranged from 25°C-30°C.
39	3.56	200	0.0178	7.65	
40	3.65	100	0.0365	36.9	3) Results of Runs 36 and 39 indicate good reproducibility.
41	4.40	145	0.0304	21.9	
42	4.00	185	0.0216	13.3	4) Inlet and outlet connections to the pump were changed from 3/8 inch I.D. tygon tubing to 1/4 inch I.D. tubing after Run 39.
43	13.20	500	0.0264	14.7	
44	13.25	750	0.0177	4.91	
45	13.40	400	0.0335	20.2	
46	2.10	100	0.021	17.0	
47	2.00	66	0.033	28.4	
48	2.00	40	0.05	79.5	
49	2.00	200	0.01	3.93	
50	2.05	135	0.0152	8.52	
51	2.03	50	0.0406	54.5	

Figure 17. Filter cake resistance (β) versus ratio of suspended solids concentration to body feed concentration (C_s/C_D) for different values of C_s

Filter aid: C535-X

Runs: 23-27 and 36-51

Water used: Iron bearing


10^8
10^7
10^6
β, ft^{-2}
SYMBOL C_S mg/l
7.5 – 8.5
3.0 – 4.0
2.0 – 2.2
13.0 – 13.5
0.01
0.1
C_S/C_D

were reported in the previous section.

Not too much work has been done in the filtration of clay waters on the pilot plant at Iowa State University. The work by Regunathan (32) on the filtration of clay water provided the only basis with which the results of the new filter could be compared. In fact, the choice of Hyflo Super-Cel as a filter aid to be used for these runs was made to simulate the conditions of Regunathan's runs in Series C and D. An exactly identical procedure was used in preparing the clay waters (Ball Clay) and a few filtration runs were made on the SSCR filter apparatus under identical conditions. There was a significant difference between the results obtained on the new apparatus and those obtained by Regunathan on the pilot plant (70 to 80 percent). Such a big difference in results can only be due to the different characteristics of the clays used. Clay samples are known to differ so much from bag to bag and from one shipment to another that unless the clay used is from the same bag or at least from the same shipment in two studies, there may not be a common basis for comparison.

Comparison between the results of constant pressure filter and SSCR filter

It was indicated as a conclusion of the constant pressure filtration study on iron bearing water that constant-

pressure techniques did not predict the results of constant rate filtration successfully. Although there did not appear to be much doubt about the validity of that conclusion on clay bearing water, it was still decided to verify this on such waters.

Several runs were made on both apparatuses (constant pressure and SSCR filter) concurrently using exactly identical suspensions. The results of these runs are reported in Table 14. The values of β-indices obtained using the SSCR filter were used to calculate the increase in time required in the successive 100 ml or 50 ml of filtrate to be obtained in constant pressure filtration for producing exactly matching results. This type of comparison was considered more convincing than comparing the values of β-indices. Reference to the data in Table 14 will prove this point. An example to demonstrate how this increase in time for a typical run was calculated is included in Appendix C.

For perfect correlation between constant rate and constant pressure results, the observed and calculated ΔT times should be equal. If the observed ΔT time is twice the calculated ΔT time, their respective values of β will differ by the same factor.

These results indicate again that no satisfactory correlation was obtained between the results obtained on the SSCR filter and the constant pressure filter.

Table 14. Comparison of observed time increment to filter successive 100 ml portions of suspension in the constant pressure runs[a] with time increment calculated from the results of SSCR runs

Run no.	C_s mg/l	C_D mg/l	$\frac{\beta}{10^6}$ ft^{-2}	Calculated ΔT in seconds	Observed ΔT in seconds	Remarks
1(a)[b]	71	90	11.6	9.17	5	Unsettled Ball Clay in tap water + Hyflo Super-Cel in Runs 1-7
2(a)	71	90	11.5	9.17	5	
3(a)	71	90	12.0	9.5	$4\frac{1}{2}$ - $5\frac{1}{2}$	
4(b)	68	150	3.65	4.7	4	
5(b)	68	105	7.95	7.2	4.5	
6(b)	68	75	16.35	10.5	6	
7(b)	89.5	107	14.1	13.0	7	
8(c)	121.2	320	19.3	33 for 25 ml	10	Settled Ball Clay in distilled water + Hyflo Super-Cel in Runs 8-19
9(c)	30.3	80	16.9	11.1	50	
10(c)	3.82	10	21.5	1.76	7	
12(c)	21.3	40	83.0	26.2	100-110	

[a]All constant pressure runs were made using a pressure of about 27 cms Hg.

[b]All runs having the same alphabet letter in the first column were made on the same day using exactly the same stock suspension.

Table 14 (Continued)

Run no.	C_s mg/l	C_D mg/l	$\frac{\beta}{10^6}$ ft^{-2}	Calculated ΔT in seconds	Observed ΔT in seconds	Remarks
14(d)	21.0	90	15.7	11.3	28	
15(d)	7.19	30	10.25	2.47	10-13	
17(e)	8.86	30	22.5	5.27	30	
19(e)	4.43	15	24.4	1.77	2-2.5	
46(f)	2.10	100	17.0	14	12-9	Iron bearing water + C535-X in Run 46
53(g)	33.9	80	3.38	2.21	4-3	
54(g)	33.9	80	2.98	2.21	3.4-1.0	Unsettled Ball Clay in distilled water + Hyflo Super-Cel in Runs 53-56
55(h)	4.95	10	8.4	0.66	2	
56(h)	79.3	160	7.2	9.03	20-6	In Run 55 and onwards modified constant pressure apparatus (Figure 4) was used to continue the runs for a longer time
61(i)	44.5	100	25.46	20	50-60	
62(i)	44.5	100	24.36	19.1	16-28	
71(j)	140	400	5.2	16.35	7-13	Ball Clay settled in distilled water + Hyflo Super-Cel in Runs 61-62 Settled Ball Clay in tap water + Hyflo Super-Cel in Run 71

If the results in Table 14 are plotted (log β versus log C_s/C_D), a series of straight lines are obtained. Only the results of runs made with the same stock suspension can reasonably be connected with a straight line. This showed clearly that the stock suspensions prepared on different days were not identical, although identical procedures were used in their preparation. This was particularly noticed on settled clay stock suspensions. The reason for the difference may be the different settling rates due to different temperatures prevailing in the laboratory on different days. In higher temperature the settling will be faster and, therefore, for the same concentration of solids, the particles will be finer and thus more resistant to filtration. This point was not investigated any further because it had no implications in this study.

The results of the runs made simultaneously using both constant pressure and constant rate techniques conclusively proved the failure of the constant pressure apparatus to predict the performance in constant rate filtration. It is hard to explain why the results of runs on some suspensions are higher in the constant pressure apparatus than the corresponding results on SSCR filter, the reverse being true on other suspensions.

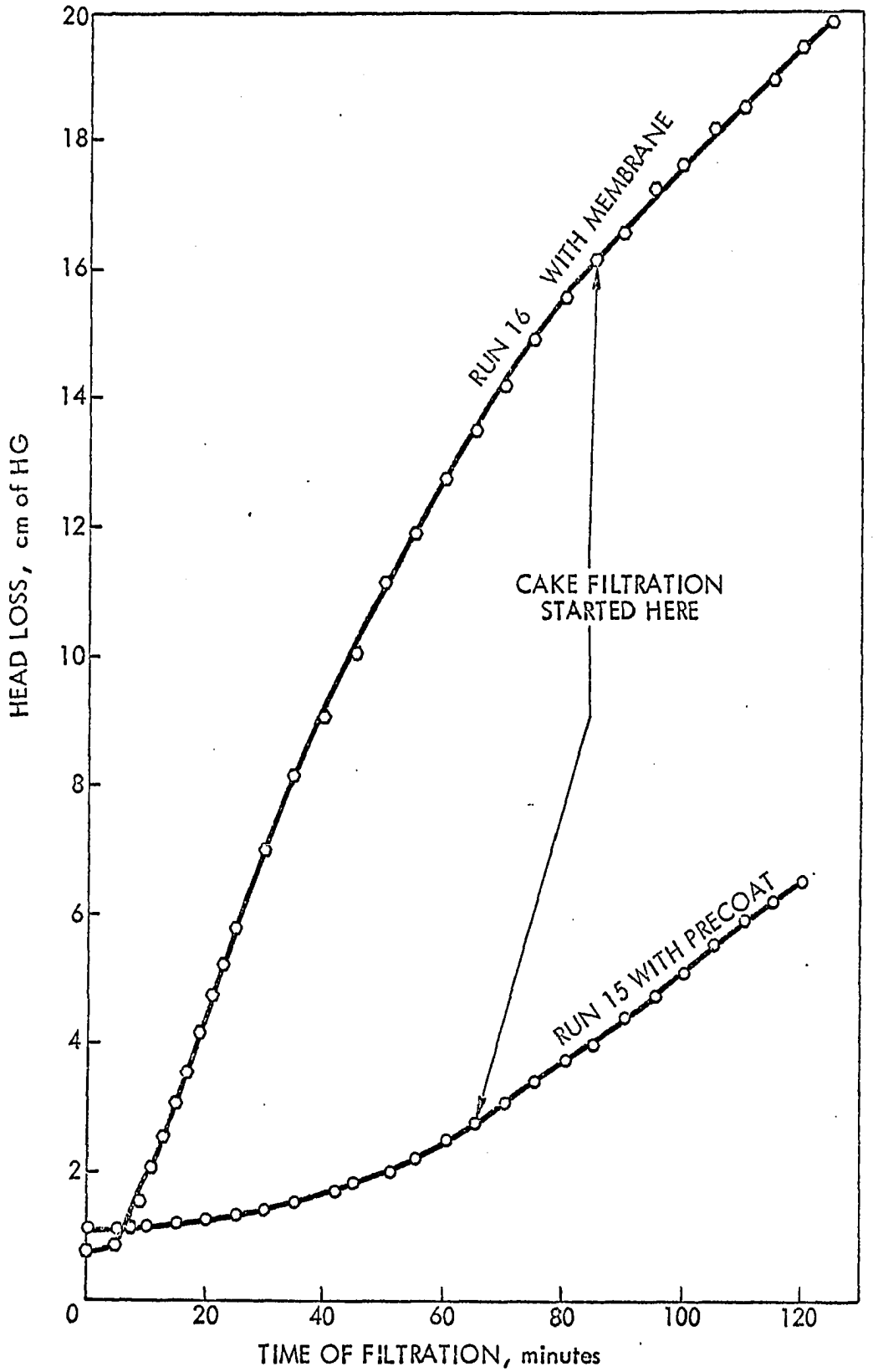
Septum plugging and cake formation

The results of two identical SSCR filter runs with one difference, a filter aid precoat was used in one run and a millipore filter membrane only was used in the other, are presented in Figure 16. It was demonstrated that the initial head loss development was different in these runs due to different cake formation characteristics on the precoat and on the millipore filter membrane. As soon as the cake had started forming, the head loss increase was almost identical in both runs.

Several duplicate runs were made on clay bearing waters to study the cake formation pattern on the precoat and on the membrane and to insure that the duplicate runs produce comparable filter cake resistance after the formation of the cake. The verification of this phenomenon was important (in the overall context of the prediction of the filter cake resistance from a simple small scale apparatus) because of the possible implications that an erroneous determination of the resistance due to the different cake formation characteristics on the particular septum used in the small scale apparatus might have on the design of the filtration installation in the field. The results of two such runs are shown in Figure 18. One of these runs (Run 15) was made using a precoat and the other run (Run 16) was made with a 0.45 micron millipore membrane only and no precoat. The values of

Figure 18. Time of filtration versus head loss on settled Ball Clay in distilled water using Hyflo Super-Cel as a filter aid

	C_s	C_D	Septum	$\beta/10^6$ ft^{-2}
Run 15	7.19 mg/l	30 mg/l	Precoat	10.25
Run 16	7.19 mg/l	30 mg/l	Membrane	13.2


HEAD LOSS, cm of HG
TIME OF FILTRATION, minutes
RUN 16 WITH MEMBRANE
CAKE FILTRATION STARTED HERE
RUN 15 WITH PRECOAT
0
2
4
6
8
10
12
14
16
18
20
20
40
60
80
100
120

the β-indices were found to be 10.25 x $10^6/ft^2$ and 13.20 x $10^6/ft^2$ for the precoat run and the membrane run respectively, results which are fairly comparable. A part of the difference between these two values can be attributed to the fact that the filtered water quality in the membrane run was slightly better than the filtrate quality from the precoat run, even after the formation of the cake. The effluent turbidity was measured several times during a run with a Hach Turbidimeter.

The removal of more clay in the membrane run would be expected to provide a higher cake resistance in the results of that run. However, once a cake was formed, the removal of clay should have been the same in both the membrane run cake and the precoat run cake. The fact that the effluent quality was consistently better (though only by 0.1-0.2 mg/l) in the membrane run would indicate that some minor amounts of solids which were being carried through the cake would be also carried through the precoat, but were being retained by the membrane. Thus, these solids, retained on the septum even during periods when a cake was formed, were also contributing to the higher value of β observed in the membrane run.

These runs confirm the earlier conclusions drawn from Figure 16:

1) The nature of cake formation on a precoat is significantly different from cake formation on a membrane.

2) The head loss at any instant is much more in a run using a membrane septum than in an identical run using precoat. The difference is essentially due to a higher initial head loss through the membrane during the cake formation period. As soon as the pores in the septum (the precoat or the membrane) are covered over with cake, the specific filter cake resistance becomes the same for a particular suspension and body concentration. Small difference, if any, existing after the formation of the cake may be attributed to the difference in filtered water quality being produced. Higher resistance will be associated with runs producing a better quality effluent.
3) Formation of a cake on a septum or a precoat can take a relatively long time. In the runs of Figure 16, this operation took about 90 minutes.
4) The values of the cake resistance would have been erroneous if the runs were not continued for a long time and the calculations based only on the early results when cake filtration had not yet started.

The prediction of filtration performance includes, in addition to head loss prediction, the determination of the quality of water that would be produced by any installation under specified filtration conditions. This is important

not only from the consumer's standpoint but also because a poor water quality can be improved by pretreatment or by changing the filter aid or its concentration. This makes it imperative that a precoat be used in SSCR filter runs so that the effluent quality is correctly predicted.

Dependence of specific resistance on concentration of solids

A study of the effect of concentration of solids on cake resistance was considered necessary in the filtration of clay bearing waters because of its importance in the design of a field installation. In other words, if this effect is absent the specific resistance will only depend, for all practical purposes on C_s/C_D for a particular water using a particular grade of a filter aid. In such a case the head loss through a filter with flat septa at any particular time during a filtration run will be directly proportional to the suspended solids concentration or body feed concentration, if C_s/C_D remains constant, since head loss is proportional to βC_D. See Equation 4 in Figure 2. Such a simple relationship will not exist, however, if the specific resistance is affected by the concentration of the solids. The runs made on iron bearing waters indicated that for this solid there was a dependence of specific resistance on the solids concentration (Figure 17). To determine whether the presence or absence of this dependence might be due to

the ionic interferences present in the tap water, it was decided to prepare clay suspensions for filtration by dilution of the clay stock solution using both tap water and distilled water. Hyflo Super-Cel was used as a filter aid in all runs.

The runs, therefore, were made using the following types of waters:

a) unsettled Ball Clay in distilled water

b) settled Ball Clay in distilled water

c) settled Ball Clay in tap water

d) unsettled Black Hills Clay in tap water.

It was mentioned earlier that the runs in any one series had to be conducted on the same day using the same stock solution because it was almost impossible to reproduce the stock solution from day to day. This limited the number of runs that could be included in any one series. It was also suspected that the stock solution might be aging continuously. In that case, the differences in the filtration characteristics from one run to the next might be due to the changing floc characteristics of the aging suspension or due to the effect of solids concentration or both. To ascertain this, identical runs were made in the morning and again in the evening on almost all types of suspensions. The results were reproducible with a reasonable degree of accuracy which led to the conclusion that the aging effects were negligible

or were nonexistent. (See results of Runs 53 and 54, 56 and 60, 61 and 62, and 75 and 78 in Table 15.) Any differences observed in the filtration characteristics in the runs of any particular series were, therefore, attributed to an effect of the concentration of solids.

The results of all runs made to study the effect of solids concentration on specific resistance are summarized in Table 15 and are shown graphically in Figures 19-22. The results of the runs in Series a and b did not provide any indication of the dependence of specific resistance on the concentration of suspended solids. The variation was essentially random. The runs in Series d using unsettled Black Hill in tap water, however, showed a definite trend of decreasing specific resistance with increasing solids concentration. Regunathan (32) made a similar observation in the filtration of an identical water on the pilot plant. Although the runs in Series c also indicated a similar trend the effect of concentration, if any, was not significant, since the extreme value was different from the mean by about 13 percent only.

In the light of the above observations, it appears that certain waters may exhibit the concentration effects and other waters may not. It might be a dangerous extrapolation, therefore, to assume that the specific resistance depends only on the ratio of concentration of solids to body feed

Table 15. Results of runs made to study the effect of concentration of solids on the specific resistance of the filter cake using clay bearing waters

Run no.	C_s mg/l	C_D mg/l	C_s/C_D	$\frac{\beta}{10^6}$ ft^{-2}	Remarks	
			Unsettled Ball Clay in distilled water (Series a)			
53	33.9	80	0.423	3.38	Morning run	Percent difference from the mean = 6.3
54	33.9	80	0.423	2.98	Evening run	
55	4.95	10	0.495	8.40		
56	79.3	160	0.495	7.20	Morning run	
57	9.9	20	0.495	8.46		Mean $\beta=8.19 \times 10^6/ft^2$. Maximum difference from the mean = 6.6 percent. Trend of variation is random.
58	158.3	320	0.495	8.38		
59	39.6	80	0.495	8.05		
60	79.3	160	0.495	8.10	Evening run	
			Settled Ball Clay in distilled water (Series b)			
61	44.5	100	0.445	25.46	Morning run	Percent difference from the mean = 2.2.
62	44.5	100	0.445	24.36	Evening run	

Table 15 (Continued)

Run no.	C_s mg/l	C_D mg/l	C_s/C_D	$\frac{\beta}{10^6}$ ft^{-2}	Remarks
64	38.4	100	0.384	28.75	Mean $\beta=24.92 \times 10^6/ft^2$. Maximum difference from the mean = 5.4 percent. Trend of variation is random.
65	153.6	400	0.384	23.40	
68	9.6	25	0.384	22.60	
		Settled Ball Clay in tap water (Series c)			
69	8.75	25	0.35	6.70	Mean $\beta=6.00 \times 10^6/ft^2$. Maximum difference from the mean = 13.3 percent. Very slight trend of decreasing resistance with increasing concentration observed, probably not significant
70	35	100	0.35	6.60	
71	140	400	0.35	5.20	
72	70	200	0.35	5.30	
		Unsettled Black Hills Clay in tap water (Series d)			
80	1.35	6.5	0.208	244.0	Mean $\beta=155.0 \times 10^6/ft^2$. Maximum difference from the mean = 57 percent. Significant trend of decreasing resistance with increasing concentration observed
77	2.68	13	0.208	165.0	
79	4.12	20	0.208	157.0	

Table 15 (Continued)

Run no.	mg/l	mg/l	C_s/C_D	$\frac{\beta}{10^6}$ ft^{-2}	Remarks
78	5.35	26	0.208	141.0	Evening run } Percent difference from the mean = 2.2.
75	5.36	26	0.208	135.0	Morning run
74	10.7	52	0.208	120.5	
76	16.1	78	0.208	104.0	

Figure 19. Variation of β-index with C_s at C_s/C_D = 0.495 in the filtration of unsettled Ball Clay in distilled water using Hyflo Super-Cel as a filter aid (Runs 55-60 in Table 15)

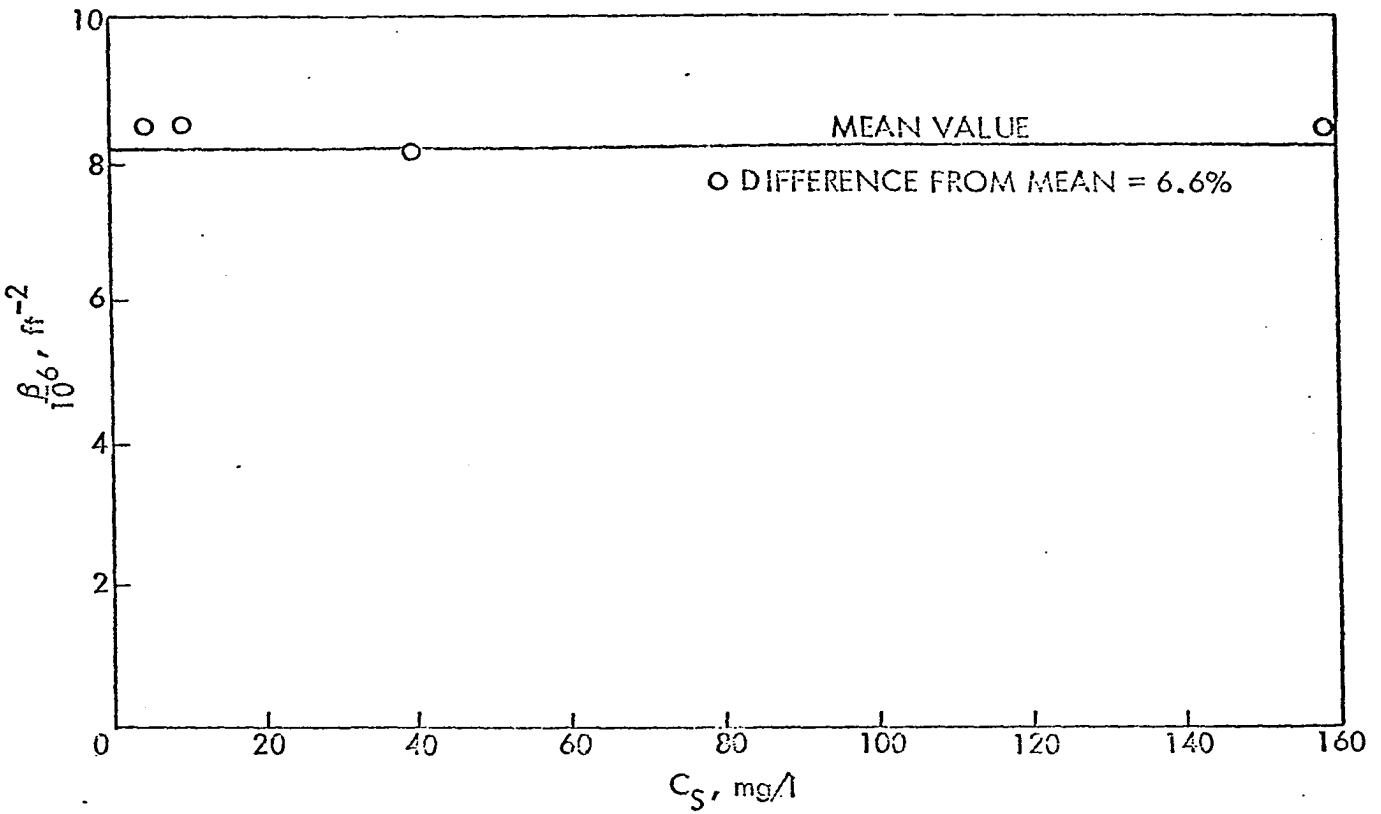

MEAN VALUE
o DIFFERENCE FROM MEAN = 6.6%
$\frac{\beta}{10^6}$, ft^{-2}
C_S, mg/l
0
2
4
6
8
10
20
40
60
80
100
120
140
160

Figure 20. Variation of β-index with C_s at C_s/C_D = 0.384 in the filtration of settled Ball Clay in distilled water using Hyflo Super-Cel as a filter aid (Runs 64-68 in Table 15)

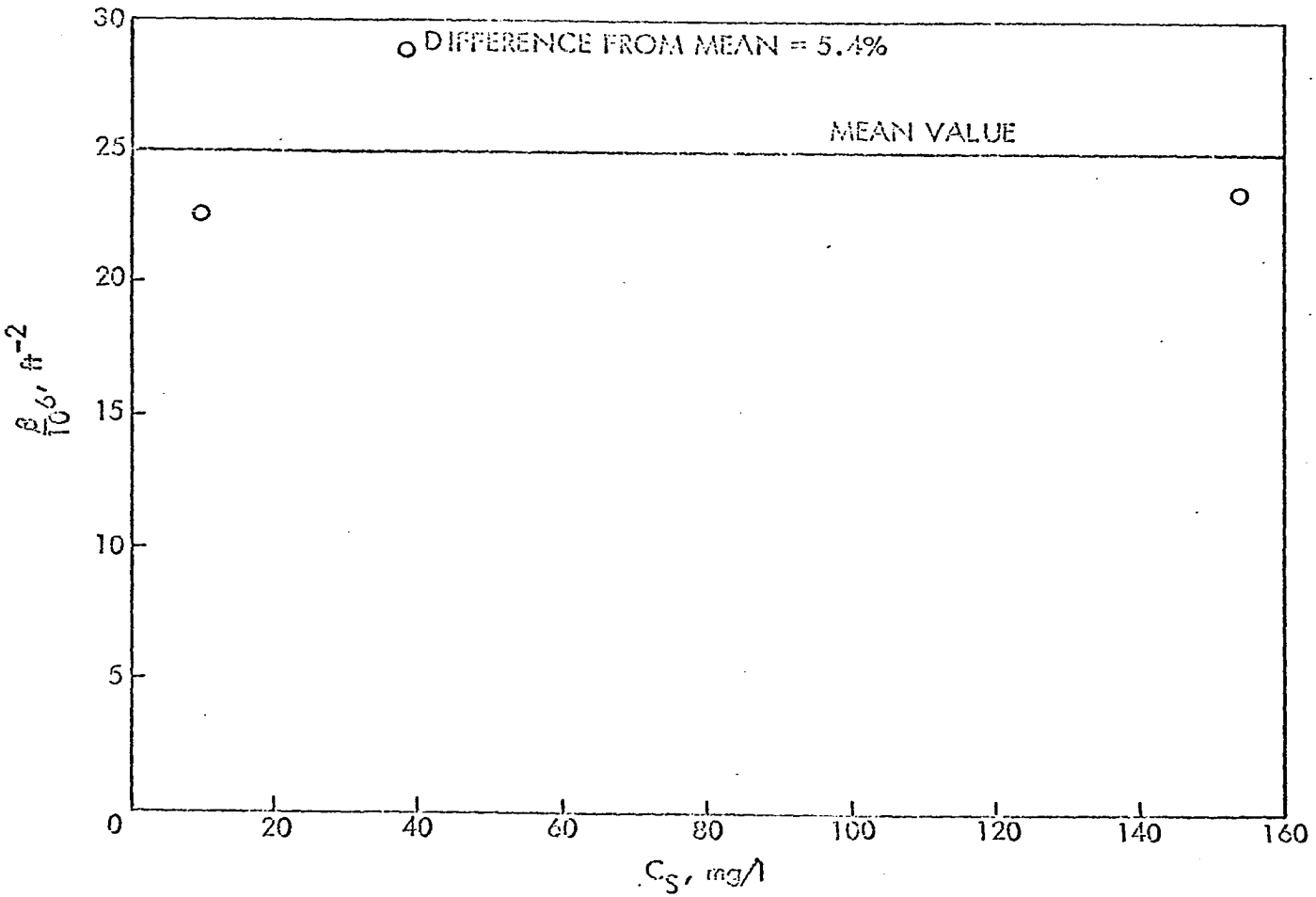

o DIFFERENCE FROM MEAN = 5.4%
MEAN VALUE
30
25
20
15
10
5
0
20
40
60
80
100
120
140
160
C_S, mg/l
$\frac{\theta}{10^6}$, ft^{-2}

Figure 21. Variation of β-index with C_s at C_s/C_D = 0.35 in the filtration of settled Ball Clay in tap water using Hyflo Super-Cel as a filter aid (Runs 69-72 in Table 15)

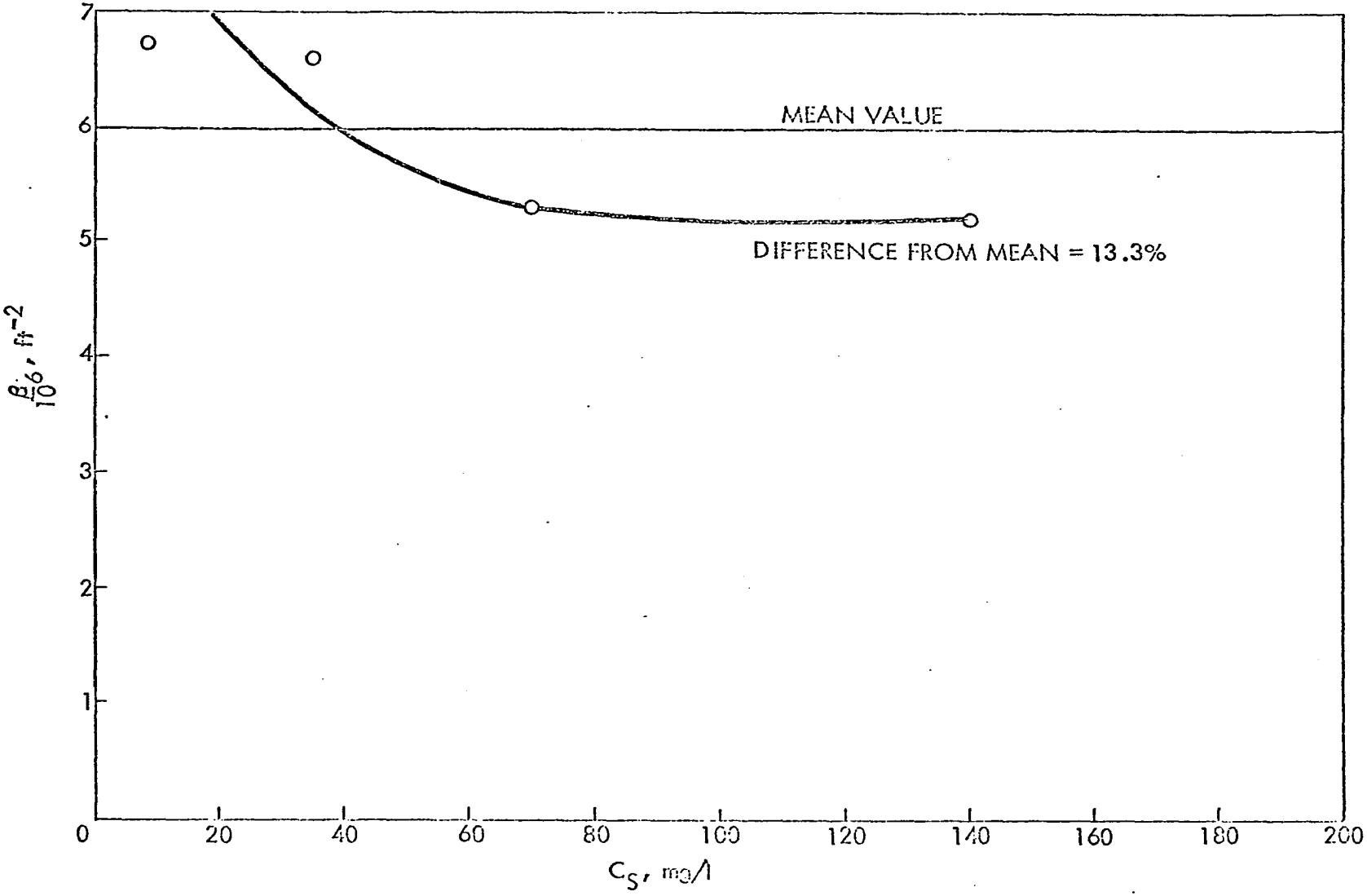
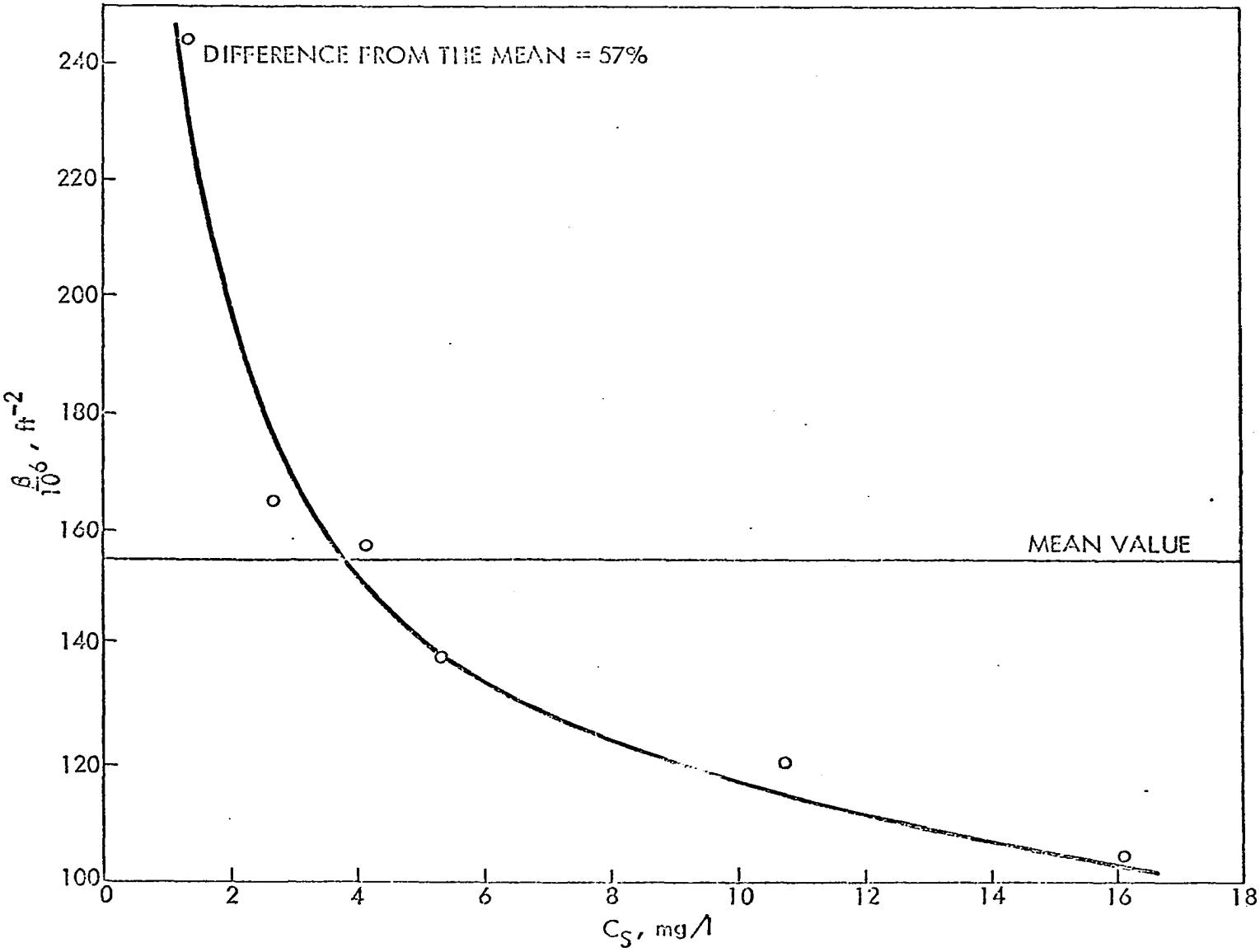

MEAN VALUE
DIFFERENCE FROM MEAN = 13.3%
$\frac{\beta_i}{10^6}$, ft^{-2}
C_S, mg/l
0
1
2
3
4
5
6
7
20
40
60
80
100
120
140
160
180
200

Figure 22. Variation of β-index with C_s at C_s/C_D = 0.208 in the filtration of unsettled Black Hill Clay in tap water using Hyflo Super-Cel as a filter aid (Runs 74-80 in Table 15)


DIFFERENCE FROM THE MEAN = 57%
MEAN VALUE
240
220
200
180
160
140
120
100
0
2
4
6
8
10
12
14
16
18
$\frac{\beta}{10^6}$, ft^{-2}
C_S, mg/l

(C_s/C_D). These results reaffirm the need to perform small scale filtration results on the actual waters likely to be encountered for successful and economical plant design.

Effect of ionic interferences in filtration

It was reported earlier that Ball Clay suspensions in distilled water were found to be 3-4 times more resistant in constant pressure filtration than exactly identical suspensions prepared in tap water. See results in Tables 8 and 9.

For example, runs 251 (Table 8) and 259 (Table 9) were made under approximately similar conditions and produced significantly different filtration resistance:

Run	Water	C_s/C_D	$\beta/10^6$ ft^{-2}
251	tap water	0.386	4.9
259	distilled water	0.367	19.0

The resistance using distilled water was thus $\frac{19}{4.9}$ or 3.9 times greater than that observed when the clay was suspended in tap water.

The results collected on the SSCR filter also showed a similar difference. Refer to the results of Runs 64-68 and 69-72 in Table 15.

For example, runs 64 and 70 (Table 15) were made under approximately similar conditions and produced significantly different filtration resistance:

Run	Water	C_s/C_D	$\beta/10^6$ ft^{-2}
64	distilled water	0.384	28.75
70	tap water	0.35	6.6

The resistance using distilled water was thus $\frac{28.75}{6.6}$ or 4.35 times greater than that observed when the clay was suspended in tap water.

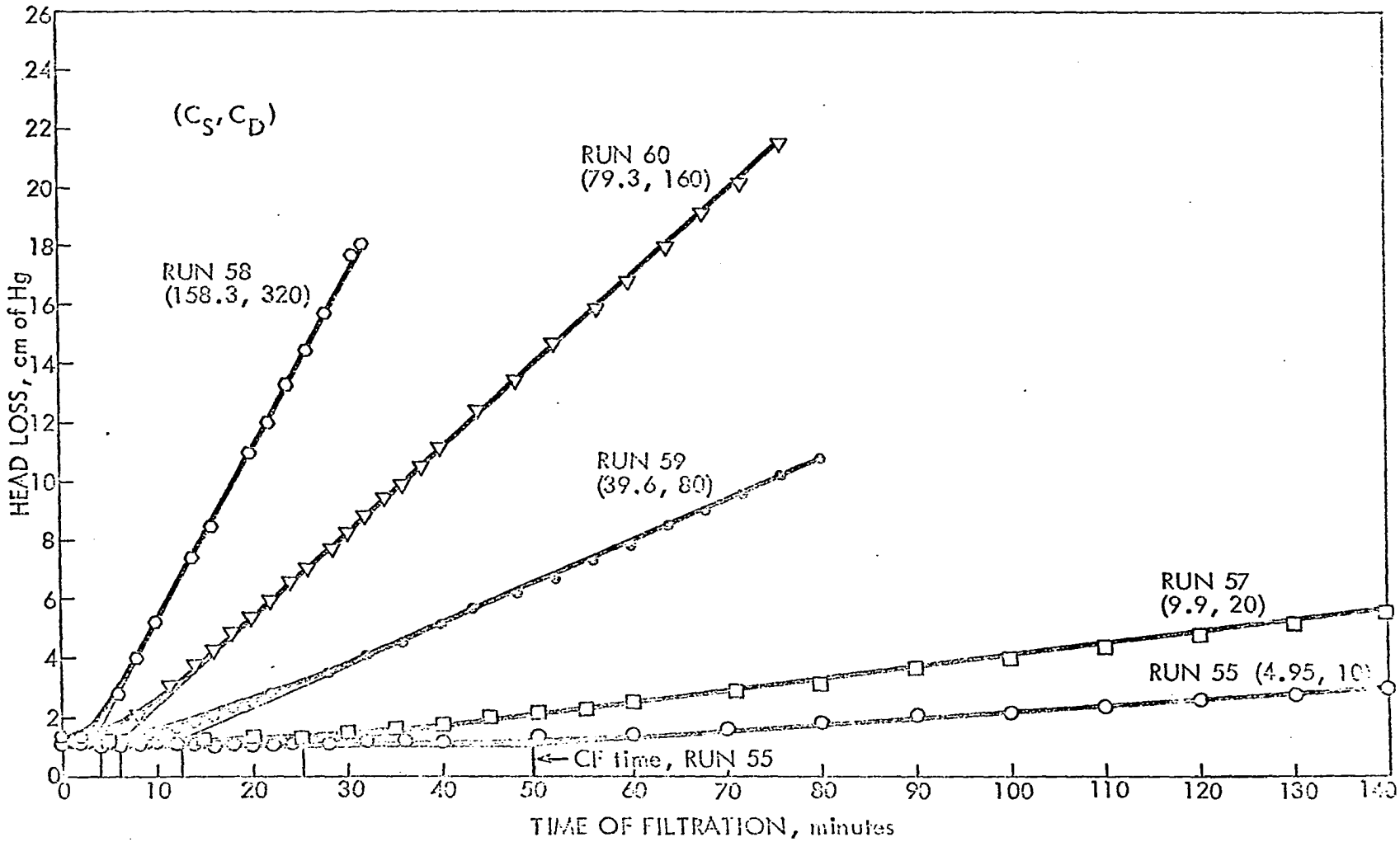
Current knowledge appears to be inadequate to explain these differences on a quantitative basis. A qualitative reasoning based on electrokinetics was advanced by several researchers (36, 40, 41) and has already been discussed.

Dependence of the septum plugging and cake formation time on the concentration of solids

While presenting the results of Figures 5 and 6 and also when the phenomenon of septum formation (precoat versus millipore membrane in Figures 16 and 18) was discussed, it was pointed out that higher concentrations of solids in the raw water reduced the time required for the septum to become uniformly covered over with cake so that solids removal was provided by the cake filtration alone and not by septum filtration. The runs made on clay bearing waters authenticated this much more conclusively. The head loss versus time of filtration data of six runs made on unsettled Ball Clay in distilled water (Series a, Table 15) are presented in Figure 23.

Figure 23. Plots of head loss versus time of filtration for runs on unsettled Ball Clay in distilled water (Series a) using Hyflo Super-Cel as a filter aid at $C_s/C_D = 0.495$ (Runs 55, 57, 58, 59, and 60 of Table 15)

The first and second figures in parenthesis next to each curve indicate the values of C_s and C_D respectively


(C_S, C_D)
RUN 60
(79.3, 160)
RUN 58
(158.3, 320)
RUN 59
(39.6, 80)
RUN 57
(9.9, 20)
RUN 55 (4.95, 10)
← CF time, RUN 55
HEAD LOSS, cm of Hg
TIME OF FILTRATION, minutes
0
2
4
6
8
10
12
14
16
18
20
22
24
26
10
20
30
40
50
60
70
80
90
100
110
120
130

The purposes of the plots in Figure 23 are to point out 1) the relative rates of head loss increase using different concentrations of solids and body feed with the ratio of the two kept constant and 2) the relative lengths of the cake formation time in such cases. Although the specific resistance was found to be almost the same within the experimental errors (see Table 15) for these runs, the rate of head loss progression (βC_D) increased with the increasing solids concentration. In fact, in this case the rate increased approximately in direct proportion to the solids concentration to maintain a constant value of specific resistance. As expected, the cake formation time (found by the intersection of the straight portion of the head loss versus time curve and a horizontal line drawn from the starting point of the curve) decreased with increasing solids concentration. See Figure 23.

It was found that the values of the cake formation time (CF time) when plotted against the concentration of solids (C_s) for a particular series of runs gave a straight line on a log plot. See Figures 24-27. Figure 25 contains two plots, one for cake formation on the millipore membrane filter and the other for cake formation on the precoat.

It is clear from Figure 25 that the millipore filter membrane permits cake formation to occur in a much shorter time than the precoat for the same solids concentration. For

example, with a solids concentration, C_s, of 10 mg/l (Figure 25), cake formation exists after 5 minutes of filtration through the membrane, but not until after 32 minutes of filtration through the precoat. One obvious reason for this occurrence is the fact that more solids are removed per unit time by the millipore membrane as compared to the precoat during the cake formation period. This is indicated by a better quality water produced by the membrane as compared to that produced by the precoat during this period. After cake filtration starts, the quality of the filtered water is essentially the same irrespective of the concentration of solids. Using the membrane as a septum, the effluent was slightly better than in an identical run with the precoat even after cake formation had occurred.

Figure 24. Cake formation (CF) time versus concentration of solids for runs in Series a of Table 15

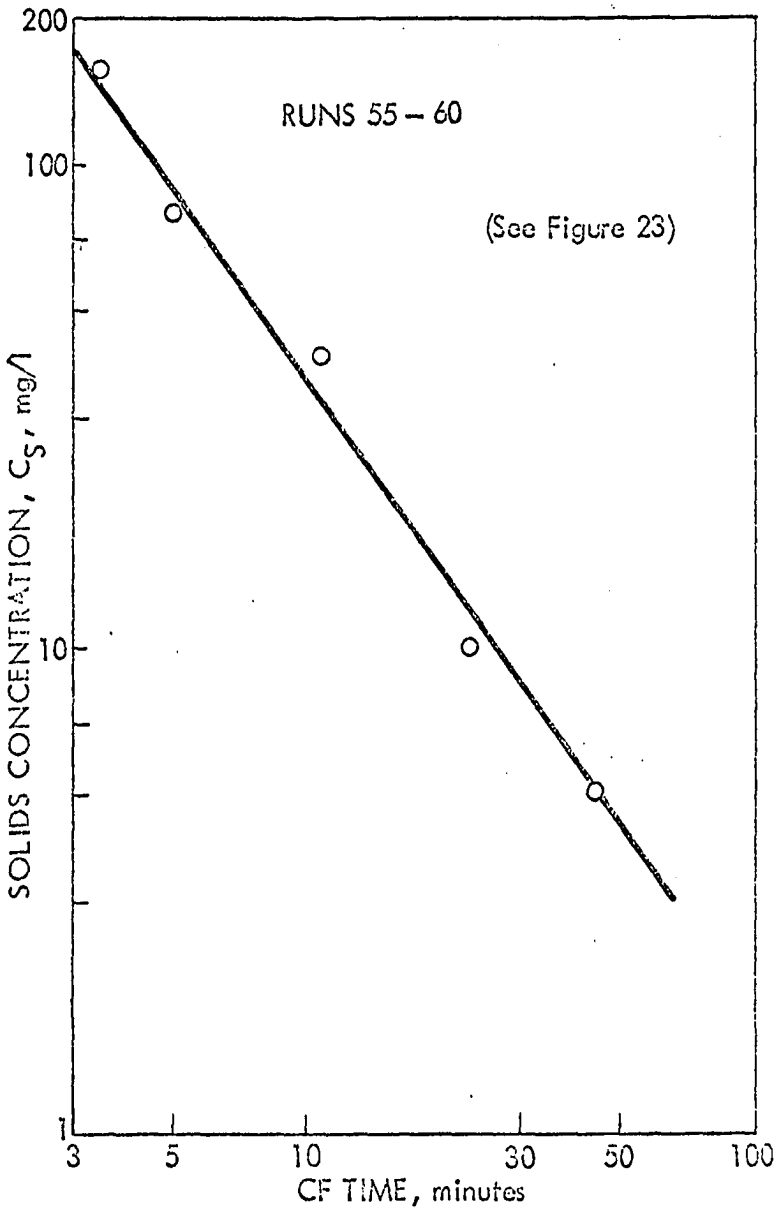

RUNS 55 – 60
(See Figure 23)
SOLIDS CONCENTRATION, C_S, mg/l
200
100
10
1
3
5
10
30
50
100
CF TIME, minutes

Figure 25. Cake formation (CF) time versus concentration of solids for runs in Series b of Table 15 and earlier runs made on the same suspension

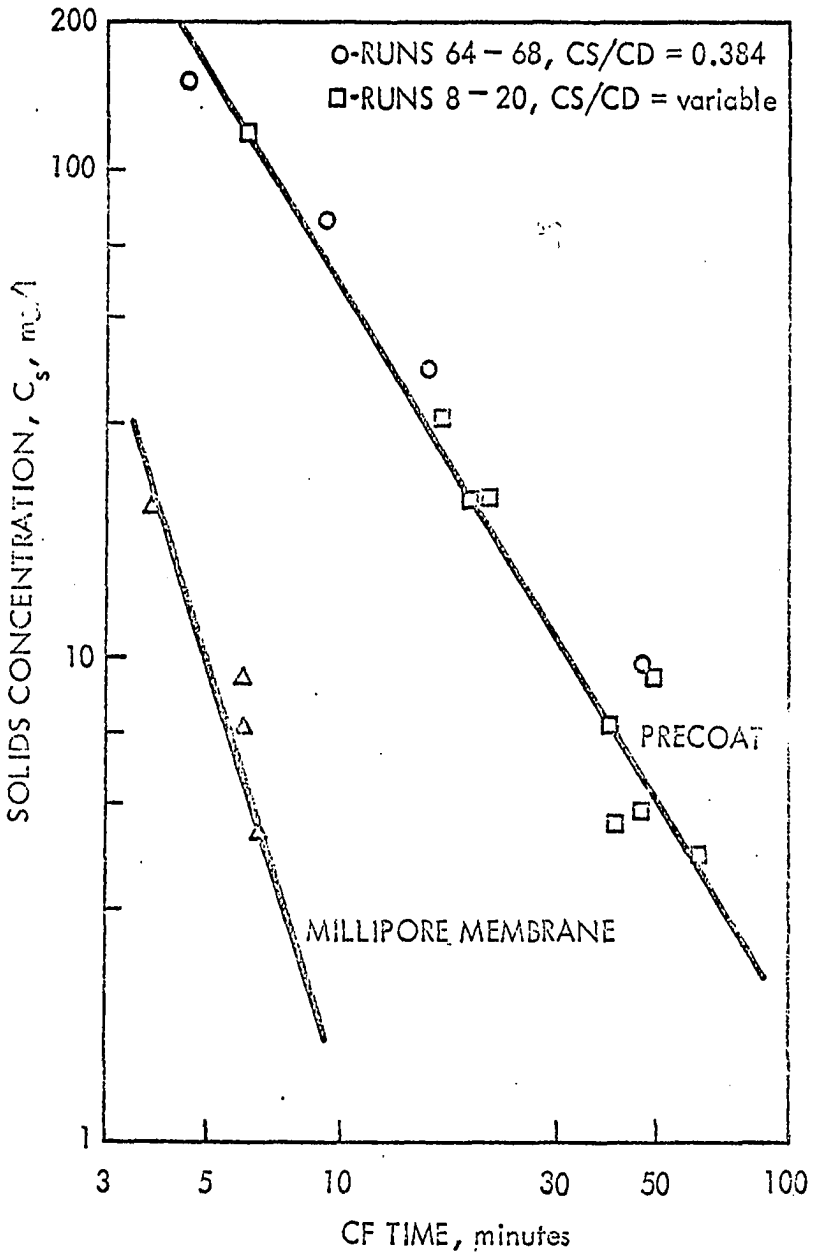

200
100
10
1
SOLIDS CONCENTRATION, C_s, mg/l
o-RUNS 64 – 68, CS/CD = 0.384
□-RUNS 8 – 20, CS/CD = variable
PRECOAT
MILLIPORE MEMBRANE
3
5
10
30
50
100
CF TIME, minutes

Figure 26. Cake formation (CF) time versus concentration of solids for runs in Series c of Table 15

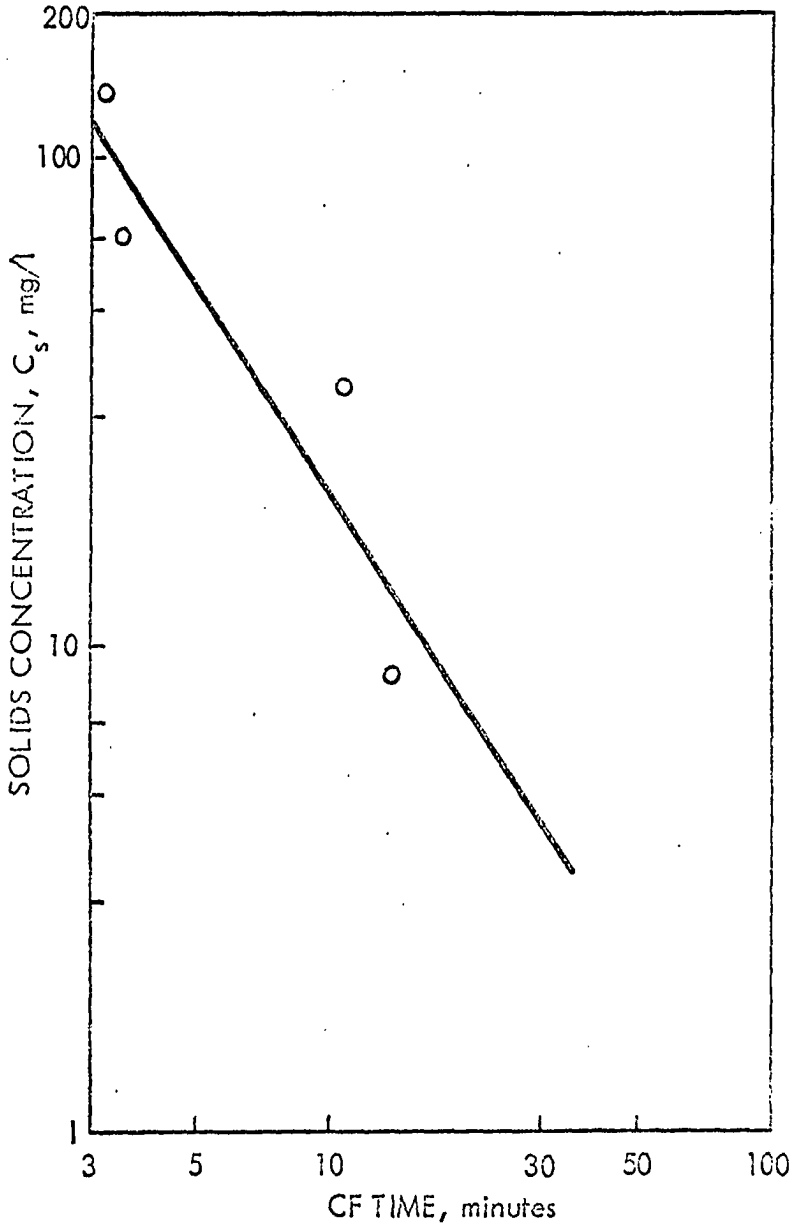
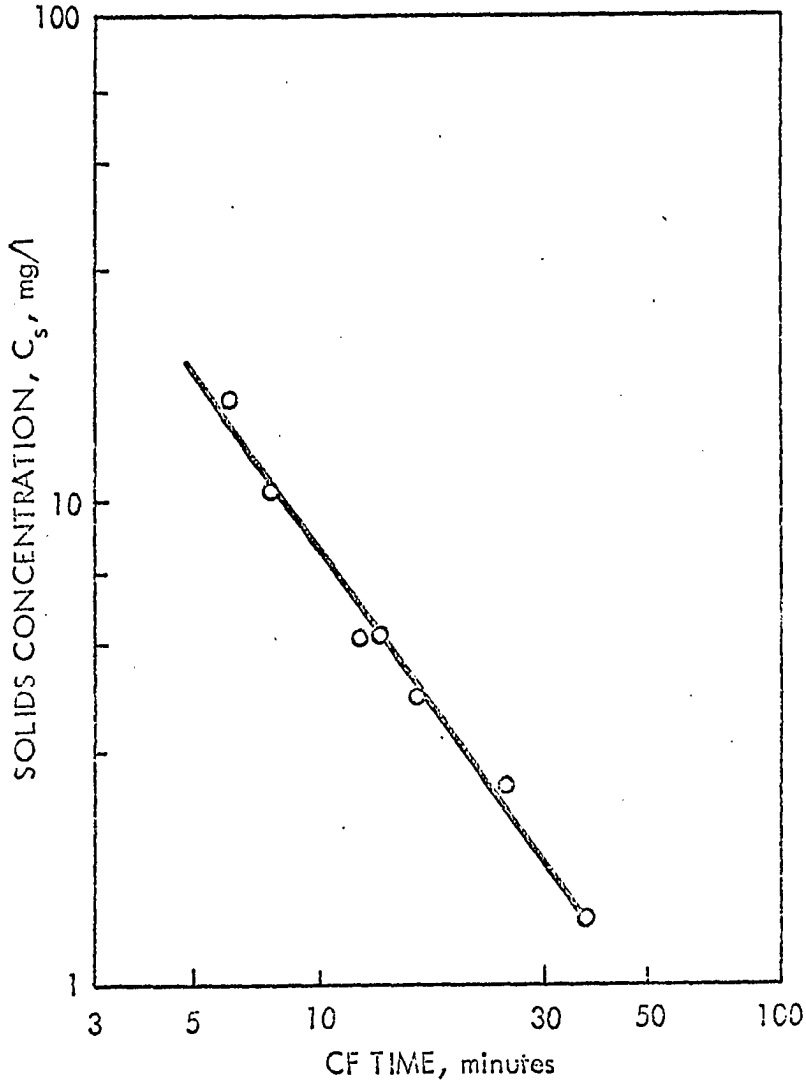

200
100
10
1
SOLIDS CONCENTRATION, C_s, mg/l
3
5
10
30
50
100
CF TIME, minutes

Figure 27. Cake formation (CF) time versus concentration of solids for runs in Series d of Table 15


100
10
1
SOLIDS CONCENTRATION, C_s, mg/l
3
5
10
30
50
100
CF TIME, minutes

DISCUSSION OF RESULTS

Theoretically filter cake resistance can be determined from either the physical characteristics of the filter aids and the solids (Equations 46 and 52) or on the basis of actual filtration tests (Equations 48, 55 and 63). The equations based on the physical characteristics have an immense theoretical merit as they provide a better insight and understanding of the filtration process and the factors that affect the process. It was realized, however, that the practical utility of these equations is severely limited because of the difficulties involved in the determination of the physical constants such as the porosity of the dirty cake, the bulk density of the cake, the mean diameter and surface area of the particles comprising the cake and so forth within a desired degree of accuracy. The specific resistance of the filter cake is extremely sensitive to the porosity of the cake, and it is highly questionable if this parameter can be determined in a cake at this period in time with reasonable accuracy. It was decided, therefore, to determine filtration resistance from actual laboratory filtration tests.

In the area of precoat filtration for production of potable water, constant rate filtration is frequently practiced. However, the first part of this study was devoted

to an investigation of the possibility of the use of constant pressure apparatus to predict the filter cake resistance obtained in constant rate filtration. The results of an earlier study (50) suggested that this might be possible. The reasons for such an attempt were both academic and practical. The constant pressure apparatus is simple to build and use, runs are not time consuming, and filter aid and test water requirements are small. It was also expected that either the success or failure of this attempt would lead to a better understanding of the filtration process and probably of the factors that affect the filtration resistance in both types of the filtration processes, i.e., constant rate filtration and constant pressure filtration. On the basis of the theory the attempt appeared to be rational and realistic.

This effort, however, was unsuccessful. The results obtained on the constant pressure filtration apparatus did not agree with the results obtained on the pilot plant in the filtration of iron bearing waters (3) using a constant rate of filtration. The results from both methods were generally 40-50 percent different. Several possible reasons for this disagreement between the two results were discussed along with the opinions and similar experiences of some researchers on this subject in the past (45, 46, 62). The main reasons for the difference in resistance results were thought to be the use of incorrect pressure for the calculation of specific

resistance, failure to account for the changing septum resistance, the magnitude of which might be different in the two types of processes, and different spatial and time distribution of porosity in the cakes laid under supposedly identical conditions maintained in these processes. For detailed discussion of these factors refer to the section starting on page 117 entitled "Discussion of Results" for the constant pressure filtration results. An interesting concept which emerged out of these results was the concept of cake formation over the septum and that either ignorance of or disregard of this phenomenon can give extremely high values of specific filter cake resistance while filtering dilute suspensions (Figures 8-10). In fact, cake resistance is not really determined as the impurities in the water and the body feed filter aid merely plug the pores of the septum and never form a cake unless the filtration run is continued for a very long period of time (Figure 6). Even when the runs were extended over a long time using the apparatus shown in Figure 4, the constant pressure results did not agree with the results obtained in constant rate filtration (Table 14), probably because of the inherent differences in the porosity distribution of the cakes and other factors discussed. This study also proved very conclusively that most of the values of specific filter cake resistance in constant pressure filtration reported by Al-Khafaji (50) are probably in error since with the dilute suspensions he was filtering, it might not have been possible to form a cake. At

the high pressures used by him in some of his filtration runs, this problem might even be worse.

It was concluded, therefore, that only a small scale laboratory apparatus working on the principle of constant rate filtration through a filter aid precoat can be used to determine the filter cake resistance applicable in the field filtration at a constant rate. Thus, a small scale constant rate filter (SSCR filter) was built and used in the second part of this study. The results obtained using this apparatus were in reasonable agreement with similar results obtained on the full scale pilot plant (Table 12), in spite of some initial problems with the apparatus and other possible sources of errors discussed earlier.

This study showed that the process of filtration and the resistance generated may be significantly affected by the ions present in the fluid being filtered. The effects on cake resistance may be explained qualitatively on the basis of electrokinetic phenomena. Both the results obtained in constant pressure filtration and constant rate filtration showed that the clay suspensions in distilled water were about 3-4 times more resistant to filter than the identical suspensions prepared in tap water. See Tables 8, 9 and 15. In preparing clay suspensions in tap water, it was observed that the supernatant obtained after 6 hours of settling was nearly free from turbidity. Identical supernatant obtained in distilled water was highly turbid. This suggested that the ions present in tap

water were causing coagulation of the solids. It is conceivable, therefore, that the reduced specific resistance of clay suspensions in tap water may be partly due to this phenomenon. Also, this observation may have an electrokinetic basis as explained earlier. Although the present knowledge of electrokinetic phenomena does not provide a quantitative basis to account for these large differences, it does explain these differences qualitatively (36). Such observations strongly reaffirm the belief that filtration behavior can be predicted successfully only by pilot plant filtration studies simulating the field conditions as closely as possible and not by the use of rigorous mathematical models alone. Mintz (63) made a similar comment:

> There are complicated interrelationships among many factors affecting the performance of rapid filters. These interrelationships vary with seasonal changes in the quality of raw water, chemical treatment, output and load changes. Therefore, it is apparent that an attempt to work out an exact mathematical description, with theoretical constants, of the filtration process to hold for any conditions of filter operation is bound to fail. Obviously, it will be always necessary to determine the parameters of the process experimentally. The task of the theory is to provide a rational experimental procedure and a rational method of working out the experimental data so as to get the results required for engineering practice.

The small-scale, constant-rate filtration apparatus (SSCR filter) not only predicted the filter cake resistance with reasonable accuracy, but it also predicted the quality of the filtered water likely to be obtained in a full scale filtration plant. In fact, a successful prediction of the

filtered water quality is a prerequisite to the successful prediction of filter cake resistance. The constant pressure filter failed to achieve this objective. This increases the usefulness of the apparatus, since the prediction of filtration performance includes, in addition to the head loss prediction, a determination in advance of the quality of the filtered water. This is important not only from the consumer's standpoint but also because the engineer can control the water quality by changing the filter aid grade or its concentration or even by suitable water pretreatment. The runs made on this apparatus in a practical situation will, therefore, provide a good basis in this direction.

It has been established authentically in this study that septum plugging and cake formation is a real and important phenomenon in precoat filtration. The results of the identical runs made on the SSCR filter, using a precoat in one run and a millipore filter membrane in the other run, confirmed the importance of the cake formation characteristics of the septum (Figures 16 and 18). This study proved conclusively that the rate of head loss increase during a run is a function of the type of septum and, of course, the nature and concentration of the solids and the filter aid in the initial period during a filtration run. However, after this cake formation period is over and the septum pores are uniformly covered with cake, the slope of head loss versus time curve will essentially be a function of the characteristics and

concentration of the suspended solids and of the body feed filter aid. Thus, it is essential to continue the filtration past this cake formation period in order to determine correctly the specific resistance of the filter cake. The values of the cake resistance determined on the basis of the results collected during the cake formation period will be erroneous. The plots of Figure 18 indicated that this period may be as high as 80-90 minutes in the filtration of clay bearing waters. It must be appreciated, therefore, that the prediction of head loss using a calculated or an assumed value of specific resistance, β-index, will be highly erroneous during this period when the pores in the septum are being covered over by the cake. In the field filtration runs, which are generally 10-15 hours long, this limitation and its effect on the head loss determination during the remainder of the run may not be important. This point will be discussed in detail subsequently.

As a result of this study, it is also evident that the effect of concentration of solids on the specific filter cake resistance can be highly significant in certain waters. See Figures 17 and 22. It may not be correct to assume, as was done by Dillingham et al. (29), that the specific resistance is primarily a function of the ratio of suspended solids concentration to the concentration of body feed (C_S/C_D). Such an assumption may lead to faulty designs. Although the specific resistance was found to be a function of only C_S/C_D for all practical purposes in some waters used

in this study, a general statement made to this effect may have serious implications. Since no satisfactory theory is available to explain the concentration effects, the earlier suggestion that a true filtration performance can only be predicted on the basis of pilot plant studies gains more evidence and support. In the design of a filtration plant the effect of the dependence of specific filter cake resistance on the concentration of solids may be self-healing in some cases. For example, an increasing concentration of solids (or body feed when C_s/C_D is constant) generally reduces the cake resistance, β-index, and thus the product of the two, $\beta\ C_D$, on which the head loss depends (Equation 4 in Figure 2) may remain the same. Several researchers in the past tried to explain the reasons for the effect of solids concentration on resistance (36, 38, 39) but the practical utility of their theories is questionable.

In this study an empirical relationship has been developed between the cake formation (CF) time and the concentration of solids while filtering a particular water with a particular filter aid used as body feed, which is:

$$t_{CF} = K\ (C_s)^n \qquad (67)$$

where

t_{CF} = cake formation time

K and n are constants, which are a function of the nature of solids, body feed and the septum.

The value of n will always be negative, which implies that t_{CF} will decrease with increasing concentration of solids. Thus, log t_{CF} versus log C_s will plot as a straight line with slope equal to n.

In earlier studies (23, 33), this phenomenon of a significant cake formation period was not recognized in precoat filtration, which was accepted unconditionally as cake filtration. The low rate of head loss increase for a long period of time in a filtration run observed in the filtration of certain types of waters (32) was attributed to an "initial dilution effect", explained earlier in this thesis. The initial dilution effect explained by Dillingham was considered to depend only on the nature and the volume of the system and the filtration rate. The length of time for which this effect would be noticed was not considered to be affected by the concentration of the suspended solids. However, the results of runs presented in Figure 23 belie this concept. The so-called initial dilution period decreases with the increasing concentration of solids in the raw water indicating clearly that some phenomenon other than mere initial dilution is taking place during this period. This study has demonstrated that the phenomenon taking place during this period is septum plugging and, therefore, the length of time before the effect of this is completed will be solids concentration dependent. In other words, the concept

that precoat filtration involves only surface or cake filtration may not be true. Depth filtration seems to precede cake filtration. The depth filtration takes place in and on the precoat during the cake formation period and negates the assumptions made in the Figure 2 equations that the head loss through a precoat remains constant throughout the filtration run.

Insufficient data is available to indicate the effect of the filter aid concentration on t_{CF}. In other words, the data in Figures 24-27 represent mainly runs made at different levels of C_s, but with the same C_s/C_D ratio. In all probability, the cake formation period will be affected both by the concentration of solids, C_s, (as shown in Figures 24-27) and also by the body feed concentration used in its filtration, as measured by the C_s/C_D ratio. The results shown in Figure 25, include runs made at several ratios of C_s/C_D. These results which form a single straight line seem to indicate that t_{CF} may be dependent on only the value of C_s of a particular type of solid. The data, however, are inadequate to justify such a conclusion.

Equation 67 is of practical value. If the head loss prediction is based only on the value of β-index determined from SSCR filter runs without any regard to the cake formation time, the predicted head loss will be much higher than will be encountered in field practice. In short runs, this

error may be significant, but in relatively longer runs the error may be negligible. Serious errors can be avoided if the value of t_{CF} is calculated on the basis of Equation 67 from the results of a few runs made on the SSCR filter and due attention is given to the t_{CF} thus calculated. See Figure 28.

This study has demonstrated the successful use of the small-scale, constant-rate filter to predict expected filtration performance in the field. The apparatus is simple to use, inexpensive to build and has been demonstrated to produce results with a good degree of reproducibility. The filter aid and test water requirements are small. A typical run may need about one gram of the filter aid and less than two gallons of the test water.

It was pointed out earlier that the optimum design of any filtration plant, whether old or new, requires in addition to the unit costs of labor, power, filter aid and the cost of the plant itself, a method to determine the filter cake resistance, β-index. The determination of this parameter is important since the computations of head loss during a filtration run can be made only if the value of this index is known for the conditions of that particular run. See the filtration equations in Figure 2. This study has provided a simple and easy method to determine the β-index.

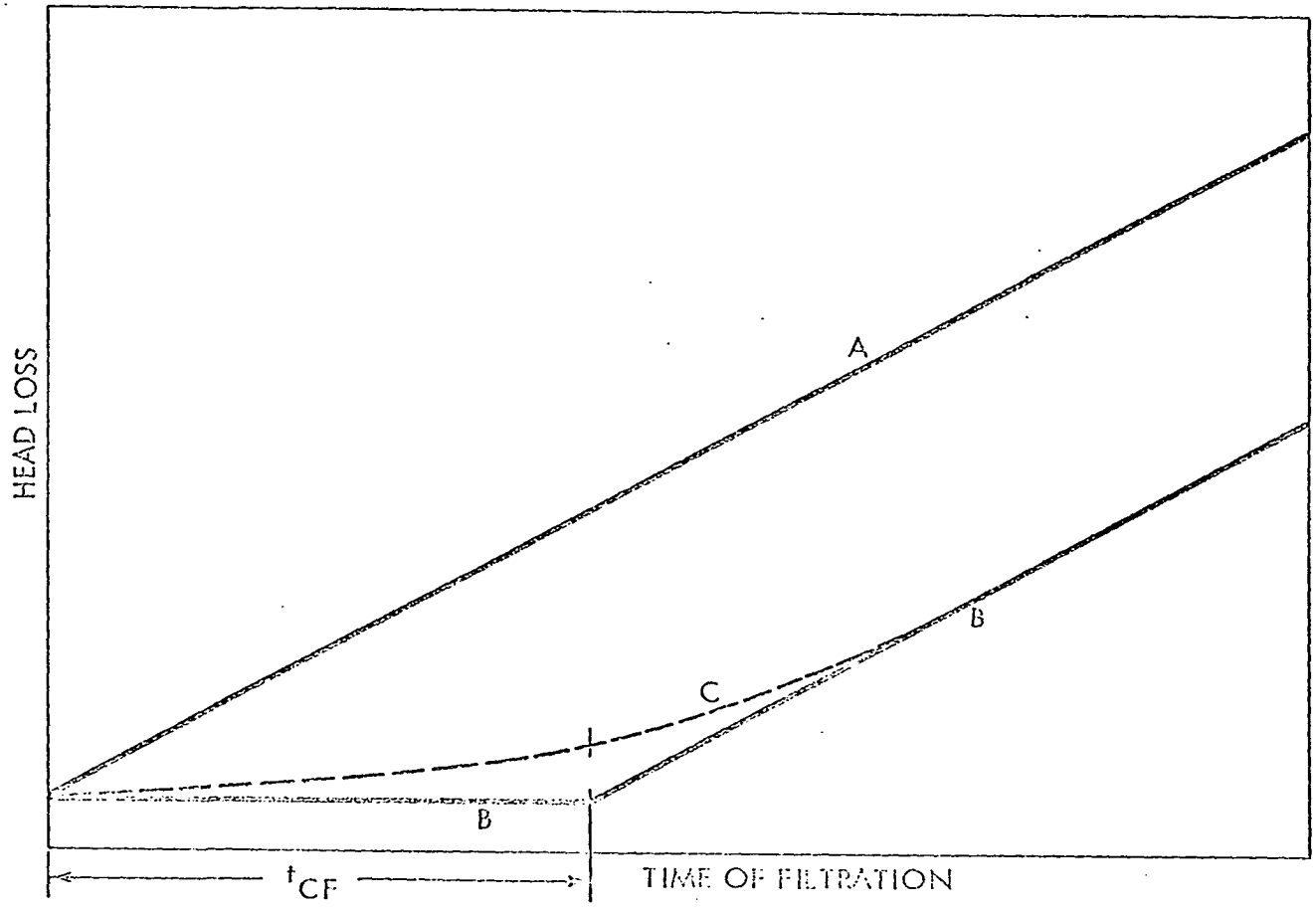
Unless a Beta prediction equation can be formulated

Figure 28. Implications of the disregard of t_{CF} on head loss prediction from a calculated value of β-index in a hypothetical run

Curve A: shows predicted head loss with no regard for t_{CF}

Curve B: shows predicted head loss after accounting for t_{CF}

Curve C: shows the actual head loss that would occur in the field


HEAD LOSS
A
B
C
B
t_{CF}
TIME OF FILTRATION

that takes into account the effect of concentration of solids on the β-index for a particular value of C_s/C_D, the use of Equations 6-8 is limited. These equations should be used with caution until it has been ascertained on the basis of previous experience or laboratory results that the concentration of solids has no effect on the value of β-index in the filtration of a particular type of water. In such a case, then, a few runs can be made on the SSCR filter at different values of C_s/C_D and the exponents of either of the Equations 6-8 determined from the results of these runs. This equation can then be used to calculate the β-index for the desired conditions of any run on the same water. The optimization of the plant is possible with manual calculations or by using a computer program like POPO. When the effect of concentration is suspected, the run on SSCR filter should be made on exactly the same conditions as in the field, such as C_s, C_D and others.

CONCLUSIONS

The results of this study are summarized below.

1) The determination of filter cake resistance from the physical characteristics of the cake is impractical because it is impossible to determine the physical constants with a reasonable degree of accuracy.
2) The results of constant rate filtration could not be predicted on the basis of the results of constant pressure filtration. The problems seem to lie in the inherent differences in the porosity distribution of the cakes laid in the two processes under supposedly identical conditions, the different amount of septum plugging occurring in the two processes and inability theoretically to account for such differences.
3) The small-scale, constant-rate filter (SSCR filter) was successful in predicting the filter cake resistance obtained in runs made using the pilot plant and iron bearing waters (3). It is concluded that this apparatus can be used successfully in the design and operation control of a field filtration plant.
4) The filtered water quality obtained in runs on the

SSCR filter matched closely with the effluent quality obtained in runs on the pilot plant. It is, therefore, expected that this filter can be used to predict not only the filtration resistance but also the expected effluent quality.

5) The Ball Clay suspensions in distilled water were found to be about 3-4 times more resistant to filtration than the corresponding suspensions prepared in tap water in both the constant rate and constant pressure filtration. This behavior may partly be due to the coagulation taking place while using tap water and also has a reasonable qualitative explanation when regarded as an electrokinetic phenomenon.

6) Precoat filtration is not a surface or cake filtration phenomenon exclusively. In a filtration run, depth filtration precedes cake filtration until the pores of the septum are uniformly covered over with the impurities in the raw water so that a cake is formed. In order to determine the true filter cake resistance, the filtration should be continued well past the cake formation time, which will depend on the nature of the septum, characteristics and concentration of solids in the raw water and perhaps also the body feed concentration.

7) Specific resistance for some waters was found to be dependent on the concentration of solids in spite

of a constant value of C_s/C_D (iron and Black Hills Clay bearing waters). In such cases, the specific resistance generally decreased with increasing concentration of solids. In the filtration of other waters cake resistance was found to be independent of the solids concentration. In a practical situation it may be incorrect to assume independence or dependence without a reasonable substantiation.

8) On the basis of the data gathered in this study the following relationship between the cake formation time (t_{CF}) and the concentration of solids (C_s) was found to hold:

$$t_{CF} = K\ (C_s)^n$$

where K and n are empirical constants that can be determined experimentally and might be a function of the nature of solids, the septum and perhaps also of the body feed. The value of n will be negative showing that t_{CF} will decrease with increasing concentration of solids.

9) The lesser rate of head loss increase in the beginning of a filtration run observed in this study and also reported in earlier studies was found to be due not only to initial dilution effect as assumed by researchers in the past, but also due to the

cake formation characteristics involved in the run.

10) A proper consideration of the cake formation time, t_{CF} can improve the head loss prediction. See Figure 28. This improvement might be significant in short runs.

RECOMMENDATIONS

The recommendations which can be made based on the results of this study are:

1. In the least cost design of a full-scale precoat filtration plant, it is essential that data be available concerning the resistance of the filter cakes composed of the solids to be removed by filtration and the body feed to be used in the full-scale plant. These data can and should be collected using the small-scale, constant rate filter described in this study.
2. The SSCR filter should be used in laboratory research to study more fully the effect of both the concentration of solids (C_s) and the concentration of body feed (C_D) on cake formation time, t_{CF}. Filtration runs should be made using several different types of solids, several filter aids, several C_s/C_D ratios, and several filtration rates so that the effects of all of these variables can be evaluated. The study should attempt to determine both the qualitative and quantitative effects of each variable so that the reasons for the effects may be determined.

3. The SSCR filter should be used in the laboratory to study extensively the effect of the solids concentration on the specific filter cake resistance. This study is extremely important in the design of a practical installation and should, therefore, be conducted to determine if this dependence can be predicted in a reasonable and a simple manner. It might be interesting to find out if the effect of solids concentration on the specific resistance is due to the different floc characteristics as a result of the changing solids concentration or whether it is, in fact, an effect due to concentration alone. The filtration of water containing some inert solids, such as glass beads, which are unlikely to change in spite of different concentrations used, through a specially designed bed, might help to answer this question.
4. The filtration equations (Figure 2) should be re-examined to take its account the concept of the cake formation time developed in this study. Revised equations may help to improve head loss prediction to be expected in a filtration run, as shown in Figure 28, by defining curve C in a more precise way.

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To my wife, Asha, thanks are specially due for her patience and for providing continued moral support and encouragement during the period when this thesis was being created.

APPENDIX A

Example Illustrating Manual Computations of β-index from the Results of a Constant Pressure Filtration Run

Data from Run 69 on iron bearing water, filter aid C535-X:

V ml	ΔV ml	ΔT sec	ΔT/ΔV sec/ml	T sec	V^1 corrected	T/V corrected
100	--	--	--	--	--	--
200	100	27.1	0.271	27.1	100	0.271
300	100	30.5	0.305	57.6	200	0.288
500	200	67.0	0.335	124.6	400	0.312
600	100	37.5	0.375	162.1	500	0.324
700	100	39.3	0.393	201.4	600	0.336
800	100	42.7	0.427	244.1	700	0.349
900	100	44.0	0.440	288.1	800	0.360
1000	100	45.7	0.457	333.8	900	0.371

Pressure = 59.5 cms Hg

C_s = 8.09 mg/l

C_D = 803 mg/l

Temperature = 23° C

Bulk density = 19.95 lbs/ft^3 [determined using the constant pressure apparatus (3]

Effluent iron concentration = 0.06 mg/l

---

[1]For plotting T/V vs V, V should be the volume of filtrate collected in time T. In 27.1 seconds, volume of filtrate is equal to 100 ml and not 200 ml. Thus, values of V have been corrected by subtracting 100 ml from the values shown in Column 1.

Slope of $\Delta T/\Delta V$ vs V from Figure 5 = 0.024×10^{-2} sec/ml^2

$= 0.024 \times 10^{-2} \times 8.018 \times 10^{8}$

$= 0.192 \times 10^{6}$ sec/ft^6

$= K$

$A^2 = (0.891)^2 \ (1/444)^2$

$= 3.829 \times 10^{-5}$ ft^4

$P = 59.5$ cms Hg

$= 59.5 \times 27.8$

$= 1652$ lb/ft^2

$\mu_{23^\circ C} = 1.9545 \times 10^{-5}$ lb sec/ft^2

$$\beta = \frac{K A^2 P}{\mu C_D}$$ (See Equation 56)

$$= \frac{0.192 \times 10^{6} \text{ sec/ft}^6 \times 3.829 \times 10^{-5} \text{ ft}^4 \times 1652 \text{ lb/ft}^2}{1.9545 \times 10^{-5} \text{ lb sec/ft}^2 \times 803}$$

$= 7.75 \times 10^{5}$/ft^2

The value obtained using the computer program was 7.97×10^{5}/ft^2 (Table 6).

Alternatively:

Plot T/V vs V; see Figure 29 for this plot.

Slope K' = 0.0115×10^{-2} sec/ml^2

$= 0.0115 \times 10^{-2} \times 8.018 \times 10^{8}$ sec/ft^6

$= 0.0922 \times 10^{6}$ sec/ft^6

$A^2 = 3.829 \times 10^{-5}$ ft^4

$P = 1652$ lb/ft^2

$$\mu_{23^\circ C} = 1.9545 \times 10^{-5} \text{ lb sec/ft}^2$$

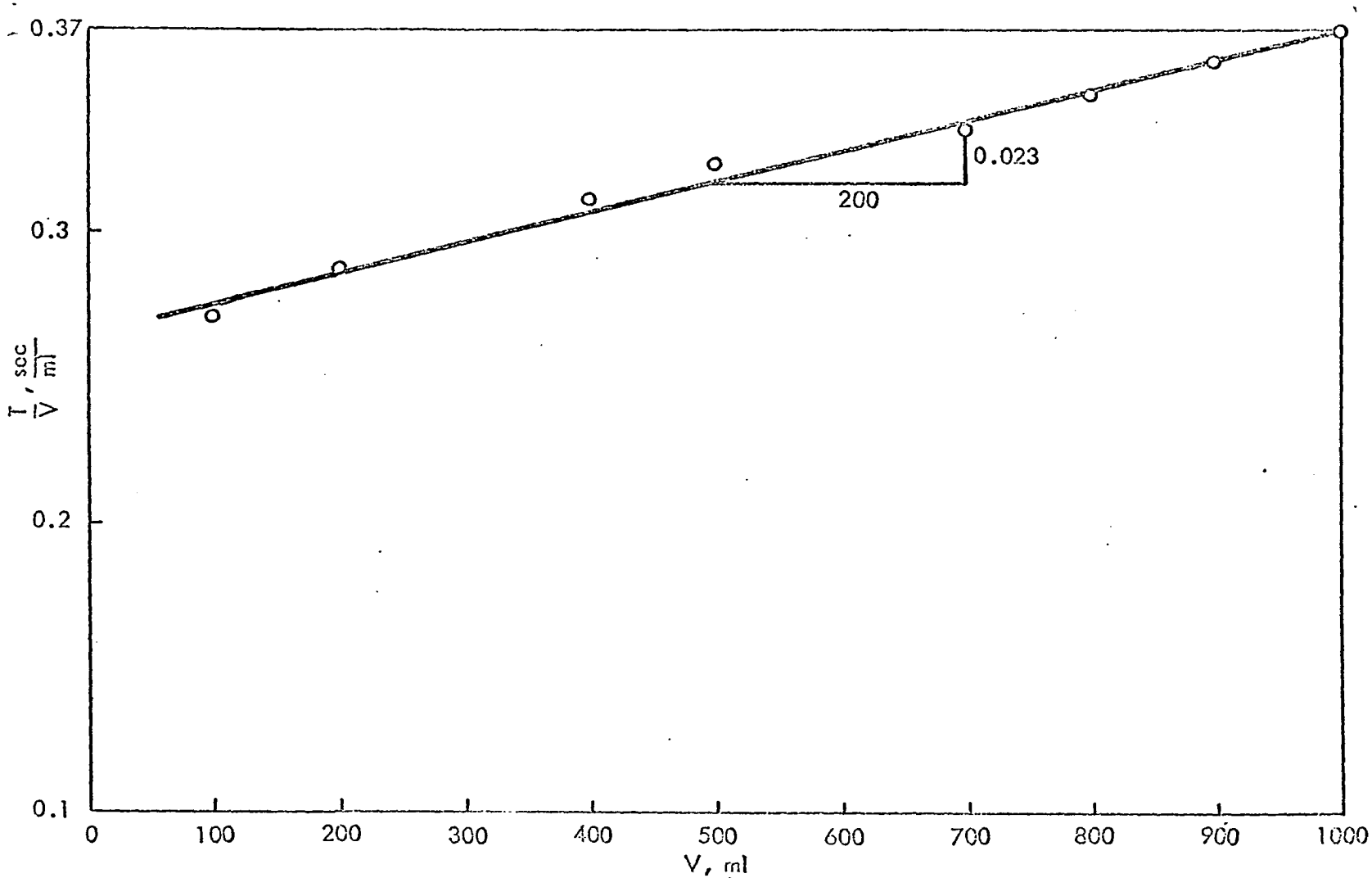
$$\beta = \frac{2 K' A^2 P}{\mu C_D} \qquad \text{(see Equation 63)}$$

$$= \frac{2 \times 0.0922 \times 10^6 \times 3.829 \times 10^{-5} \times 1652}{1.9545 \times 10^{-5} \times 803}$$

$$= 7.45 \times 10^5/\text{ft}^2$$

Thus, the β-index can be calculated either way.

Figure 29. Plot of T/V versus V for the data of Run 69 made on constant pressure filtration apparatus


0.37
0.3
0.2
0.1
T/V, sec/ml
0.023
200
0
100
200
300
400
500
600
700
800
900
1000
V, ml

APPENDIX B

Example Illustrating Manual Computation of β-index from the Results of a Constant Rate Filtration Run

Run no. 3:

Head loss in cms of Hg	Minutes elapsed since start of run
1.0	0
1.2	5
1.8	10
2.9	15
4.0	20
5.0	25
6.2	30
7.5	35
8.7	40
10.3	45
11.4	50
12.8	55
14.2	60

Filtration rate, q = 1.05 gallons per minute per ft^2

Average temperature = 24.5° C

Precoat = 0.15 lb/ft^2

Filter aid used = Hyflo Super-Cel

Suspension used = unsettled Ball Clay in unfiltered tap water

C_s = 71 mg/l

C_D = 90 mg/l

Calculations:

See Equation 4 in Figure 2.

$H_C = \sigma X$

$= q^2 \nu \beta C_{D/g} X$

$= q^2 \nu \beta C_{D/g} t$ if $X = t$

Thus, slope of the plot between H_C and t will be equal to $q^2 \nu \beta C_D/g$. Assuming that the head loss through the precoat and the septum are negligible throughout the run, the total head loss measured at any time during the filtration run can be considered to occur through the cake.

Refer to Figure 15 for the plot of H_C versus t.

The slope was measured as 0.26 cm Hg/minute (of the straight line following cake formation)

= 0.26/2.54 inches of Hg/minute

= 0.26/2.54 x 1.05 ft of water/minute

= (0.26 x 1.05/2.54) x 60 ft of water/hour

= 6.45 ft of water/hour

$= q^2 \nu \beta C_D/g$

Therefore,

$\beta = 6.45\ g/q^2 \nu C_D$

$g = 32.2\ ft/sec^2$

$= 32.2 \times 3600 \times 3600 \text{ ft/hr}^2$

$= 417.31 \times 10^6 \text{ ft/hr}^2$

$q = 1.05 \text{ gpm/ft}^2$

$= 1.05 \text{ gal/min ft}^2 \times 60 \text{ min/1.0 hr} \times 1.0 \text{ ft}^3/7.48 \text{ gal}$

$= 8.41 \text{ ft/hr}$

$C_D = 90.0 \text{ mg/l}$

$\nu = 0.972 \times 10^{-5} \text{ ft}^2/\text{sec at } 24.5^\circ \text{ C}$

$= 0.972 \times 10^{-5} \times 3600 \text{ ft}^2/\text{hr}$

$= 3500 \times 10^{-5} \text{ ft}^2/\text{hr}$

$= 3.5 \times 10^{-2} \text{ ft}^2/\text{hr}$

Now

$$\beta = 6.45 \, g/q^2 \, \nu \, C_D$$

$$= \frac{6.45 \text{ ft/hr} \times 417.31 \times 10^6 \text{ ft/hr}^2}{8.41 \times 8.41 \text{ ft}^2/\text{hr}^2 \times 3.5 \times 10^{-2} \text{ ft}^2/\text{hr} \times 90}$$

$$= 12.0 \times 10^6/\text{ft}^2$$

Alternatively:

Refer to Equations 48 and 21 in the Literature Review chapter.

$$\frac{Pt}{V/A} = R' \, w \, \mu \, (V/A) + \mu \, R_f$$

and

$$R' \, \gamma_w/10^6 = \beta$$

Thus, slope of the plot of $\frac{Pt}{V/A}$ versus $\frac{V}{A}$ will be equal to $R' \, w \, \mu$, from which R' can be calculated.

The values of $\frac{Pt}{V/A}$ and $\frac{V}{A}$ were calculated for Run 3 and are listed in Table 16.

$$\text{Slope} = 0.16/_{1.75}\ \text{hr/ft} = 0.0915\ \text{hr/ft}$$

$$= 0.0915 \times 62.4\ \text{lb hr/ft}^4$$

$$= 5.71\ \text{lb hr/ft}^4$$

$$w = 90 \times 10^{-3} \times 6.243 \times 10^{-2}\ \text{lb/ft}^3$$

$$= 562 \times 10^{-5}\ \text{lb/ft}^3$$

$$\mu = 1.880 \times 10^{-5}\ \text{lb sec/ft}^2$$

$$= 0.522 \times 10^{-8}\ \text{lb hr/ft}^2$$

$$\therefore R' = \frac{5.71\ \text{lb hr}}{\text{ft}^4 \times 562 \times 10^{-5}\ \frac{\text{lb}}{\text{ft}^3} \times 0.0522 \times 10^{-8}\ \frac{\text{lb hr}}{\text{ft}^2}}$$

$$= 0.0195 \times 10^{13}\ \text{ft/lb}$$

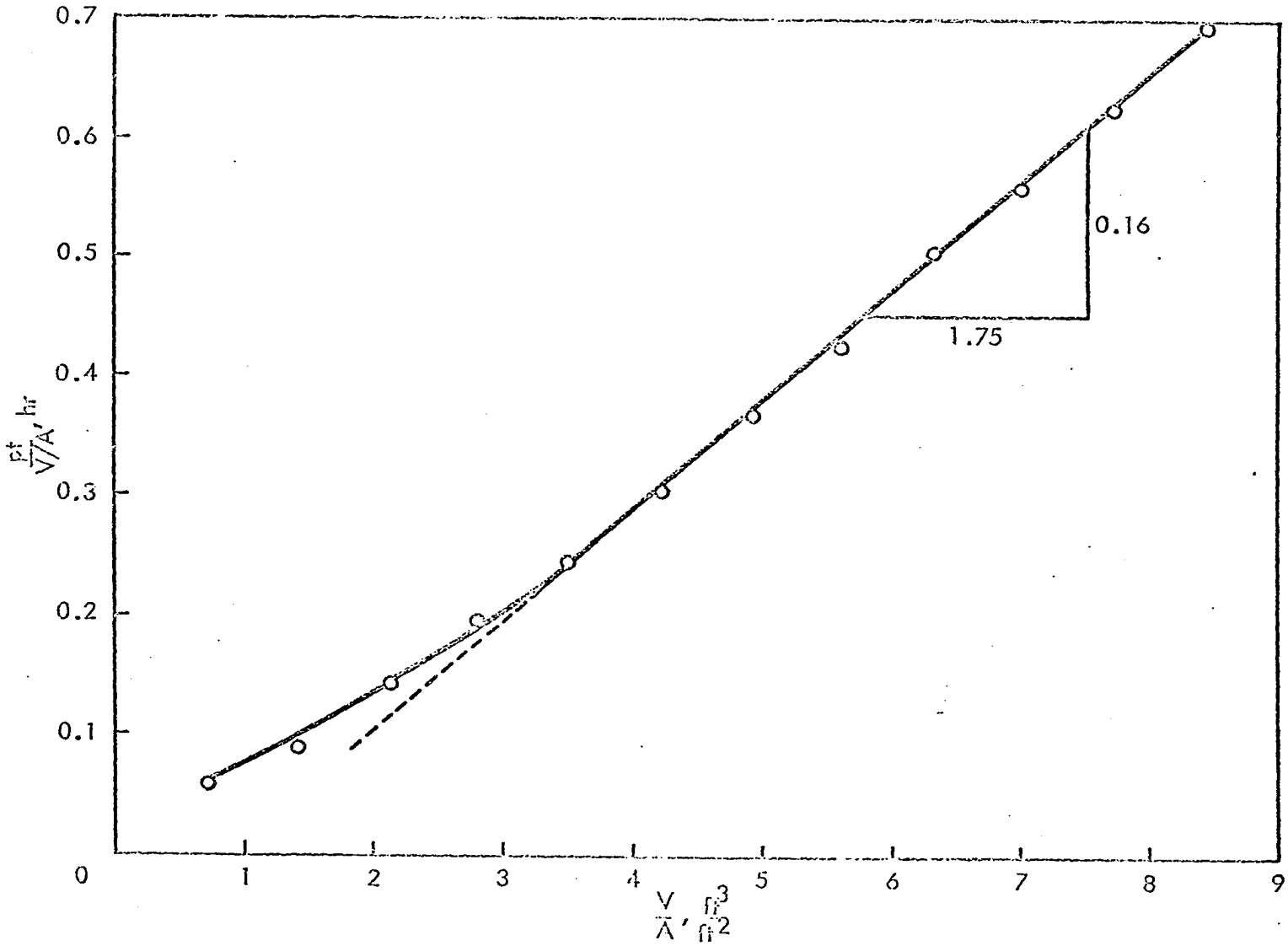
$$\beta = 0.0195 \times 10^{13} \times 62.4/10^6 = 12.15 \times 10^6/\text{ft}^2$$

Table 16. Calculations of $\frac{Pt}{V/A}$ and $\frac{V}{A}$ for the results of Run 3

t minutes	t hours	P[a] ft water	Pt ft hr	V/A gallons/ft^2	V/A ft^3/ft^2	$\frac{Pt}{V/A}$ hr
0	0	0.413	0	0	0	--
5	0.083	0.496	0.0412	5.25	0.702	0.0586
10	0.167	0.745	0.1243	10.50	1.404	0.0885
15	0.250	1.200	0.3000	15.75	2.106	0.1425
20	0.333	1.652	0.5500	21.00	2.808	0.1960
25	0.417	2.065	0.8600	26.25	3.510	0.2450
30	0.500	2.561	1.2810	31.50	4.212	0.3042
35	0.583	3.100	1.8100	36.75	4.914	0.3680
40	0.667	3.600	2.4000	42.00	5.616	0.4270
45	0.750	4.260	3.2000	47.25	6.318	0.5070
50	0.833	4.715	3.9300	52.50	7.020	0.5600
55	0.917	5.300	4.8550	57.75	7.722	0.6290
60	1.000	5.875	5.8750	63.00	8.424	0.6970

[a] 1 cm Hg = 0.413 ft water.

Figure 30. Plot of $\frac{Pt}{V/A}$ versus $\frac{V}{A}$ for the results of Run 3 on SSCR filter


0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
1
2
3
4
5
6
7
8
9
0.16
1.75
$\frac{ft}{V/A}$, hr
$\frac{V}{A}$, ft^3/ft^2

APPENDIX C

Example Illustrating a Sample Calculation for ΔT (seconds) in Column 5 of Table 14

Run no. 55:

Water used = unsettled Ball Clay in distilled water

Filter aid used = Hyflo Super-Cel

C_s = 4.95 mg/l

C_D = 10 mg/l

Average temperature in constant pressure run = 26° C

β calculated from the results of run made on SSCR filter

$= 8.4 \times 10^6/ft^2$

Pressure used in constant pressure run = 27.2 cms Hg

Note: The constant pressure run was made on the modified apparatus (Figure 4) and thus the run was continued for a long time in which about four liters of suspension was filtered.

Calculations:

Assuming that an identical run made on the constant pressure apparatus will produce a cake with exactly the same specific resistance as observed on the SSCR filter. β should be equal to $8.4 \times 10^6/ft^2$.

Using Equation 63:

$$\beta = \frac{2\ K'\ A^2\ P}{\mu\ C_D}$$ where K' is slope of T/V versus V

$$= \frac{K\ A^2\ P}{\mu\ C_D}$$ where K is slope of $\Delta T/\Delta V$ versus V

since 2 K' = K (Equation 60A)

$A^2 = (0.891/144)^2 = 3.829 \times 10^{-5}\ ft^4$

$P = 27.2$ cms Hg

$= 27.2 \times 27.8\ lb/ft^2$

$= 755\ lb/ft^2$

$\mu = 1.825 \times 10^{-5}$ lb sec/ft^2

$C_D = 10$ mg/l

Thus,

$$8.4 \times 10^6/ft^2 = \frac{K \times 3.829 \times 10^{-5}\ ft^4 \times 755\ lb/ft^2}{1.825 \times 10^{-5}\ lb\ sec/ft^2 \times 10}$$

Therefore,

$$K = \frac{8.4 \times 10^6/ft^2 \times 1.825 \times 10^{-5}\ lb\ sec/ft^2 \times 10}{3.829 \times 10^{-5}\ ft^4 \times 755\ lb/ft^2}$$

$$= 0.53 \times 10^5\ sec/ft^6$$

$$= \frac{0.53 \times 10^5}{8.018 \times 1.8}\ \frac{sec}{ml^2}$$

$$= 0.0657 \times 10^{-3}\ \frac{sec}{ml^2}$$

= slope of $\Delta T/\Delta V$ versus V

or

$$0.0657 \times 10^{-3}\ \frac{sec}{ml^2} = \frac{d(\Delta T/\Delta V)}{d\ V}$$

If the increment of volume, for which ΔT is required is 100 ml, then

$$0.0657 \times 10^{-3} \frac{\text{sec}}{\text{ml}^2} = \frac{\Delta T/100 \text{ ml}}{100 \text{ ml}}$$

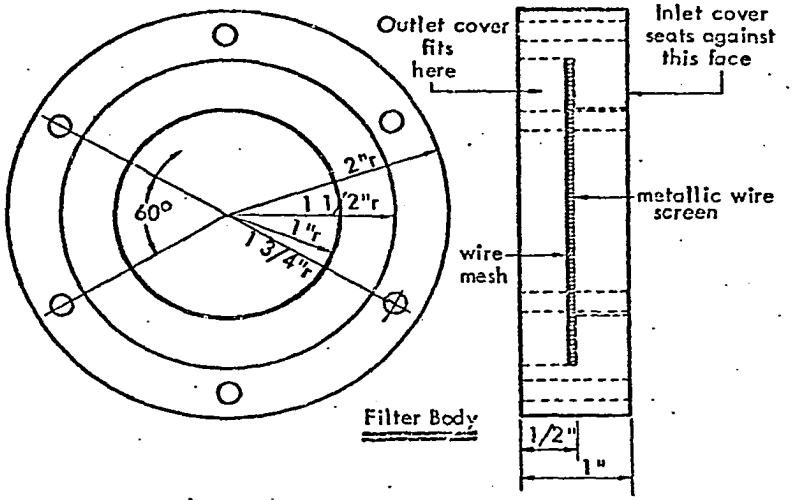
Solving

$\Delta T = 0.657$ second

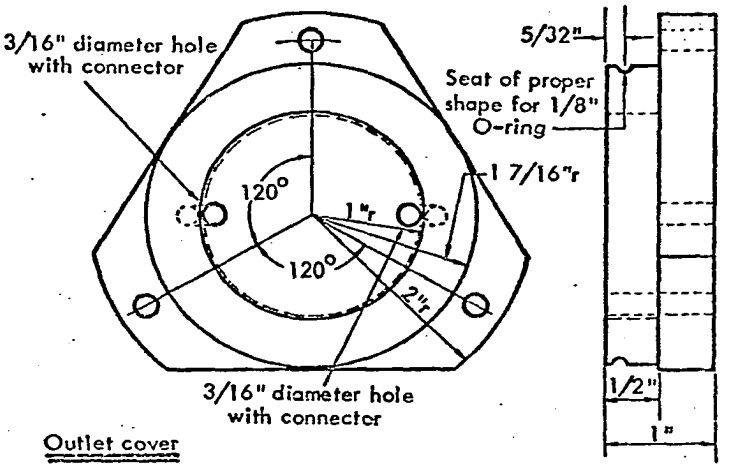
In other words, if the time required for the filtration of first 100 ml suspension is 10 seconds, it should be 10.657 seconds for filtration of the next 100 ml suspension and 11.314 seconds for the next 100 ml suspension and so forth.

APPENDIX D

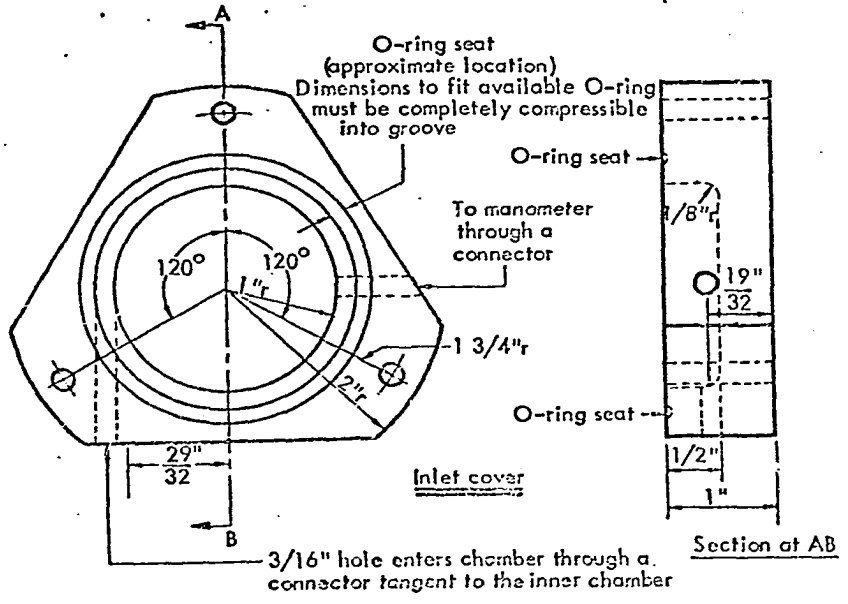
Figure 31. Working drawings of the filter assembly in SSCR apparatus



Filter Body

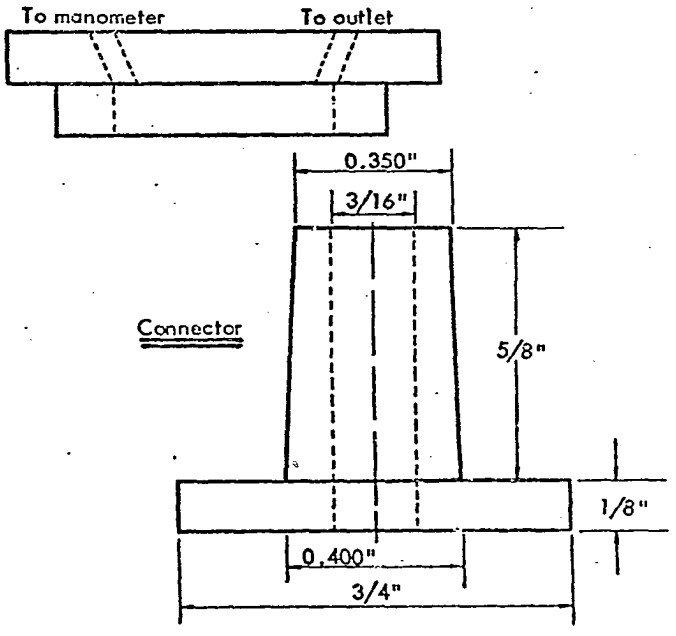


Outlet cover



Inlet cover

Section at AB



Connector